

**A Unified Methodology for the Management of  
Time-Dependent Information in Aeronautical Systems**

by

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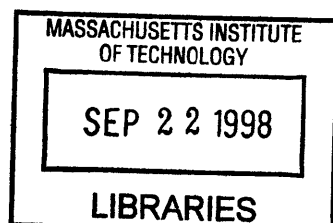
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**Aero**



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## **ABSTRACT**

The introduction of aeronautical datalinks as an alternate communication modality will enable direct connectivity between airborne and ground-based computers. The resulting ability of automation to accomplish communication tasks without human intervention will make feasible a variety of decision-support systems that require the communication of time-varying information. Historically, the exchange of information via voice communication has been managed through heuristically developed manual procedures. This thesis proposes a generalized approach to designing processes, both automatic systems and manual procedures, for efficiently managing the communication of time-varying information, when communication resources are constrained.

To answer questions about how frequently or under what conditions aging information should be updated, a novel model of time-dependent information value, which combines elements of classic information value theory with estimation techniques, is developed for a class of “proceduralized” decision problems.

The value of information is measured by its effect on the ability of the decision maker to estimate relevant state variables in the context of the proceduralized decision problem. The periodic rate at which information must be updated is shown to depend on the rate at which the expected error in the decision maker’s state estimate becomes significant in the context of the proceduralized decision problem, as well as the acceptable latency in the decision maker detecting unexpected events. Derivative or intent information is shown to effect the required update rate by changing the rate at which the expected error grows. A cooperative information management environment is shown to reduce the required update rate by providing efficient detection of unexpected events.

Application of the model is demonstrated through two case studies. The first examines the measurement and dissemination of airport surface observations, to support an early decision whether or not to divert to an alternate airport. The second examines the rate at which aircraft state information should be transmitted to maintain separation between proximate aircraft in an Automatic Dependent Surveillance (ADS) environment. Results provide insight into the time-dependence of information and its implications on information management processes.

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To my parents

– *Te Amo*

Jill – rrrribbit! Thank you for being a very quiet talking-frog. I could not have finished this without your love over the past year, and all of the hugs from Cinnamon and Chester.

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# Notation

$P[\bullet]$	The probability that the event $\bullet$ will occur
$P[\bullet \star]$	The probability that the event $\bullet$ will occur, given the event $\star$ has occurred
$f_{x(t) I}$	The probability density function for the state $x$ at time $t$ , based on available information $I$
$p_{r I}$	The probability mass function for the discrete random variable $r$ , given event $I$ has occurred
$E[\star]$	The expected value of $\star$
$V$	Expected information value
$V x$	Conditional information value, given measurement $x$ is received
$C$	Expected cost
$R$	Expected uncertainty cost
$I$	A piece of information
$x(t)$	A state variable $x$ at time $t$
$\hat{x}(t)$	An estimate of the state variable $x$ at time $t$
$\mathbf{x}(t)$	A vector of state variables at time $t$
$t$	A point in time
$r_i$	A region in the state space
$a$	An action or decision
$\theta$	A threshold surface
$\kappa$	Payoff matrix
$\Gamma$	Context matrix

“As I hammered in the last bolt and staggered over the rim, it was not at all clear to me who was the conqueror and who was the conquered. I do recall that El Cap seemed to be in much better condition than I was.”

Warren Harding – on completing the first ascent of the Nose, El Capitan, Yosemite Valley, 1958.



# Chapter 1

## Introduction

New technologies (e.g., datalink communication, satellite-based navigation, and remote weather sensing) make feasible a variety of pilot and controller decision-aiding systems that require the measurement and communication of time-varying information. Several examples are collision-avoidance alerting systems, conflict detection automation for traffic management, and the display of graphical weather information in aircraft cockpits to support flight planning. The need to manage time-dependent information for decision-aiding or alerting also exists in application areas outside transportation systems, such as battlefield management, investment decision-making, the operation of power generation plants, and the control of manufacturing processes.

Historically, processes for managing time-dependent information (e.g., system design decisions concerning how frequently information should be measured or communicated) have been developed ad hoc. This thesis presents a generalized approach for making decisions about when information should be updated. Several questions

are addressed. How does the age of information affect its ability to support a decision problem? If one or more new measurements may be taken, how does the value of the information represented by those measurements vary with the time at which the measurements are taken? When – how frequently or under what conditions – should measurements be taken to support a decision problem?

This thesis focuses on information applied to proceduralized decision problems (i.e., decision problems in which an established procedure or rule specifies the correct decision or action as a function of a set of relevant state variables) since they represent a common class of decision problems, the decision-making agent's decision policy is known from the procedure, and the cost structure is well defined.

A novel time-dependent information value model is presented as a tool for studying and designing processes to manage time-dependent information, when measurement or communication resources are limited. The model is studied in the context of operationally interesting problems to gain insight into the nature of time-dependence in information value and its implication to information management.

## **1.1 Motivation**

Prior to the introduction of aeronautical datalinks (e.g., ACARS, Mode-S, and JTIDS), most aviation communication was accomplished verbally, requiring that all information be buffered at each end of a radio-frequency voice channel by a human (Figure 1.1). The prevalence of submissions to NASA's Aviation Safety and

Reporting System (ASRS) citing failures in information transfer suggests that an entirely voice-based system may be unable to support evolving aviation communication requirements (Billings & Cheaney, 1981). To motivate applying automation to manage the delivery of information in a datalink environment, Lee and Lozito (1989) observed that one-fourth of 14,000 ASRS filings involved problems in air/ground information transfer.

A recent incident, in which pilots failed to receive timely surface pressure information with which to set the aircraft's barometric altimeter, exemplifies the inefficiency in the existing voice/procedural approach to disseminating necessary information. On November 12, 1995, American Airlines Flight 1572 flew through the tops of trees on a ridge 3 miles short of the Runway 15 threshold at Bradley International Airport. At the time of the accident, a low pressure system was rapidly moving through the area – as pressure drops, an uncorrected barometric altimeter will display an altitude higher than the aircraft's true altitude. The aircraft was more than 300 feet lower than the altitude which the aircraft instruments were displaying. The pilots failed to receive a sufficiently current altimeter setting with which to correct this error.

Figure 1.1 illustrates the relative placement of humans and computers in voice and datalink communication. The introduction of aeronautical datalinks as an alternate communication modality has enabled direct connectivity between airborne and ground-based computers. The resulting ability of automation to accomplish communication tasks without human intervention makes feasible a variety of datalink applications (e.g., graphical presentation of ground-measured weather information on aircraft flight decks) that promise to improve aircraft safety and operating

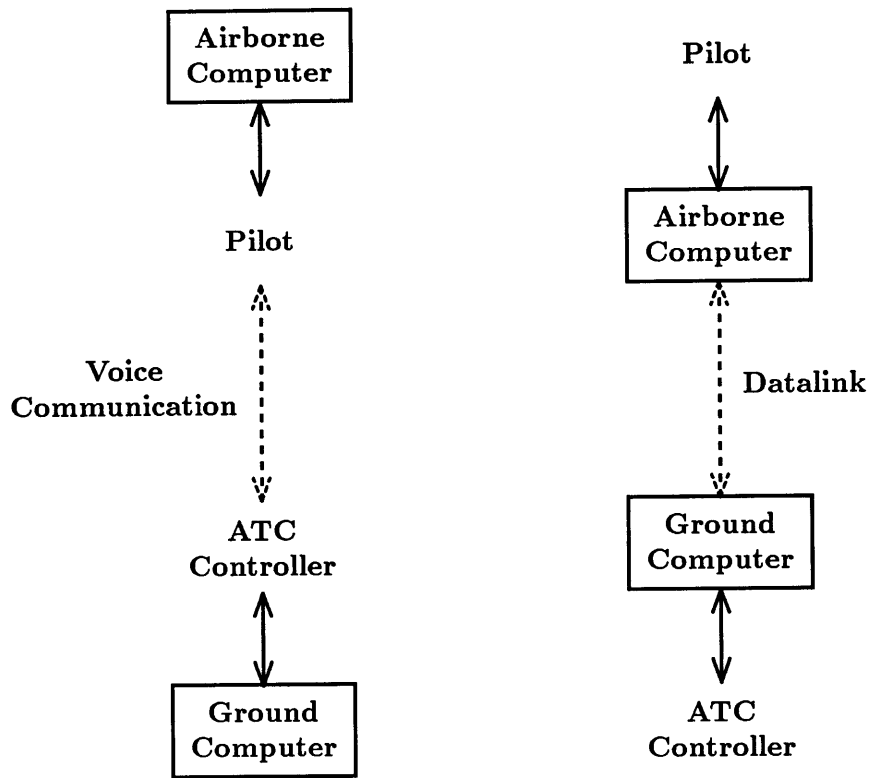
efficiency by allowing pilots to make better informed decisions and by reducing nonfunctional pilot workload.<sup>1</sup> Moreover, the ability to automate certain communication tasks will foster an approach to air/ground communication that exploits the complementary characteristics of voice and datalink, allowing the exchange of increased amounts of information in readily usable formats, reducing transmission and interpretation errors, and relieving the saturation of VHF voice frequencies, which currently impedes efficient message transfer during peak traffic periods in busy terminal areas (FAA, 1995).

Without careful study, however, the overall impact of introducing new datalink-based cockpit systems on the flight crew, as well as on aircraft safety and operating efficiency, is unpredictable, since these systems may add tasks or alter pilot responsibilities. Wiener (1988; 1989), Curry (1985), Wiener and Curry (1980), Billings (1991), and Hopkin (1991), among others, have observed the uncertain impact of automation in aeronautical systems. Because the impact of datalink communication has not yet received significant attention, there is a need for additional research in this area.

The manner in which the flight crew interacts with the information and the datalink – the pilots’ interface to the accessed information as well as to the process of accessing that information via the datalink – will strongly affect the overall impact of datalink-based cockpit systems. Although the pilots’ interface to the information (e.g., display format, data fusion, and decision aiding) has been studied extensively in both aviation and other contexts, the manner in which the pilots interact with

---

<sup>1</sup> Tasks resulting from a particular human/machine interface to a cockpit system that do not directly contribute to situation awareness constitute “nonfunctional” workload.



**Figure 1.1. The relative placement of humans and computers in voice and datalink communication.**

the datalink – how information transfer is managed – has received less attention.

In voice communication, a consequence of the information being buffered by humans is that information is managed manually, typically through a procedural approach. This method is feasible since both the voice format and the manual protocol constrain the amount of information that may be exchanged. In datalink communication, only channel bandwidth and message cost will limit the amount of information that may be exchanged between ground stations and aircraft, resulting in a proliferation of the available information. Furthermore, some datalink-

based systems such as Automatic Dependent Surveillance, that require automatic exchange of information, isolate the information users – pilots and controllers – from the communication process. Consequently, aeronautical communication in a datalink environment will require a novel solution to managing information transfer, in which humans and automation share responsibility. Numerous paradigms, representing various levels of autonomy, may be proposed for the manner in which pilots and automation interact to cooperatively control the flow of information, à la Sheridan (1992).

The increased information density and the need to partially automate information management require an approach to designing information management processes, both automatic systems and manual procedures, that is more rigorous than the ad hoc method historically used. This thesis considers an approach that employs a formal measure of information value as a quantitative foundation for making decisions concerning seeking and using information.

The concepts of the quantity of information contained in a measurement or message and the value which information contributes to decision making have been well established for certain classes of problems. Chapter 2 reviews a selection of relevant literature on the concepts of information and information value. However, in dynamic environments, such as the aviation task domain, the state of the world varies with time. Consequently, as a piece of information, which describes the condition of a state at the time it was measured, ages, it provides less of an indication of the current condition of the state and the value with respect to a decision problem decreases. Existing time-invariant theories of information value are incapable of modeling this relationship between information value and information age.

## 1.2 Information versus Measurements

The term *information* is used throughout this thesis. A definition is appropriate. According to Shannon's (1949) definition, the *information* contained in a measurement, or message, is a characteristic of that measurement which depends on the a priori probability of receiving it, relative to the probabilities of receiving other possible measurements. An alternate paradigm for the relationship between information and measurements, popular in the "Command, Control, Communication, and Intelligence" (C<sup>3</sup>I) literature, is based on the extent to which measurements are processed: *data* is the result of processing a *measured signal*; *information* is the product of filtering data; and *knowledge* is the result of a decision maker receiving and comprehending information.

The objective of this thesis is to develop methods for efficiently managing the collection, communication, or display of information, for decision-aiding applications. *Information* is defined, broadly, as anything which can be collected and communicated or displayed, and receipt of which by a decision maker may support a decision-making process. A *measurement* is information collected by taking a physical observation or sample of a state variable at a particular time. Measurements commonly contain uncertainty about the true condition of the state variable, due to observation or sensor errors. A *state variable* is a dimension along which a condition of the world, or a subset of interest, may be measured.

Although the discussions in this thesis are frequently in terms of "measuring a state variable," since this action is more intuitive than "collecting information," nothing about the approach limits its applicability to measured information. Note, however, that the application of the information will always be to improve

a model of the state variables on which a proceduralized decision depends. Therefore, this thesis is concerned with information, either measured or non-measured, that contributes to the modeling of a set of relevant state variables. Examples of non-measured information are a checklist or a page from an emergency procedures manual. Non-measured information is typically, although not always, constant over the period of time relevant to the decision. This thesis is principally interested in information that is time-varying, since repeated updates are required to maintain a certain level of confidence in the state model. As an issue of semantics, information about a state variable that is constructed from other information, rather than directly measured (e.g., a derivative that is estimated from position measurements), will be said to be calculated, rather than measured.

### **1.3 Thesis Overview**

The technical approach taken by this thesis is to develop a novel model for valuing information that captures the time-dependencies of the information and decision, and to study this model in the context of operationally interesting problems. To begin, Chapter 2 reviews much of the progress that has previously been made in the area of valuing information. Given this foundation, Chapter 3 introduces the model of time-dependent information value, in a general form. Chapters 4 and 5 demonstrate the application of the model through two case studies. Chapter 4 considers the problem of managing the measurement and dissemination of airport surface observations, in the context of supporting a pilot, en-route to an airport



at which conditions are near the approach minima, in making an early decision whether or not to divert to an alternate airport. Chapter 5 considers the problem of managing surveillance information for traffic management and collision avoidance applications. Chapter 6 summarizes the results of the thesis.

“I don’t climb mountains simply to vanquish their summits. What would be the point of that? I place myself voluntarily into dangerous situations to learn to face my fears and doubts, my innermost feelings.”

Reinhold Messner – first person to climb all fourteen mountains over 8000 meters.

## Chapter 2

# Information and Information Value

### 2.1 Classical Literature

Although the mathematical foundation of information theory may be traced to the concept of entropy in thermodynamics and statistical mechanics (Kullback, 1959), Fisher (1925) is credited with introducing, to the theory of statistical estimation, a measure of the *information* that a piece of data supplies about an unknown parameter. Shannon (1949) and Wiener (1948) independently proposed logarithmic measures of information, in the context of communication theory, stimulating the substantial study on the subject that has followed.<sup>1</sup>

As interpreted by Sheridan (1995), Shannon, who was studying communication in the presence of noise, defined information as:

---

<sup>1</sup> Kullback (1959) offers a thorough history of the early literature on information theory, discussing the dichotomy between its application to communication theory and experimental design.

“the reduction in uncertainty about the state of an event after a message has been sent relative to the uncertainty about the state of the event before the message was sent.”

Information quantity is measured in *bits*, where one bit equals the amount of information required to decide between two equally likely alternatives (Sanders & McCormick, 1993). Mathematically, the information conveyed by a message indicating that an event  $x_i$  has occurred is proportional to the logarithm of the reciprocal of the probability  $P(x_i)$  that the event would occur. Consequently, a message identifying the occurrence of an event that was expected represents little information.

If one of  $N$  events  $x_1, x_2, \dots, x_N$  might have occurred, where the a priori likelihoods are known to be  $P(x_1), P(x_2), \dots, P(x_N)$ , respectively, the average information  $H$  contained in a message that describes which event did occur equals the weighted sum of the information inherent in each event occurring.

$$H = \sum_{i=1}^N P(x_i) \log_2 \frac{1}{P(x_i)} \quad (2.1)$$

Therefore, the information contained in the message is maximum, for a given number of possible events, when the distribution of the probabilities of occurrence is uniform across the events (Shannon & Weaver, 1949). In summary, the Shannon definition of information measures the average uncertainty before a message was sent, with respect to which event occurred, and the uncertainty reducing capacity of the message.

Howard (1966) noted that Shannon’s definition of information involves only the probabilities of events occurring, without considering the consequences of events

occurring, and, therefore, fails to capture the significance of uncertainty to a decision maker. Howard introduced the theory of *information value* to study, in the context of decision making, the relation between the stochastic nature of uncertainty and the economic impact of uncertainty. Sheridan (1995) explained Howard's definition of information value as:

“the difference in what one can gain by action taken knowing the state of the event relative to what one can gain by action taken without such knowledge.”

Howard summarized the applicability of his information value definition in the following way.

“Placing a value on the reduction of uncertainty is the first step in experimental design, for only when we know what it is worth to reduce uncertainty do we have a basis for allocating our resources in experimentation designed to reduce the uncertainty.”

Howard (1967) observed that the expected profit resulting from a decision is an inadequate measure on which to base decisions. As an alternative to specifying utility curves that express risk preference, Howard characterized the nature of the risk that results from uncertainty through the probability density function, describing the likelihood that various profits would be realized.

Howard also observed that the value of information may be measured either with or without knowledge of the information content, the *expected* value of the information being the relevant measure in the later case. To describe uncertainty in the measurement of information value, Howard used a probability distribution for the value of information.

## 2.2 Decision Theory

### 2.2.1 Overview

If a decision maker knows the true state of nature, the outcome of selecting each action in that state, and the criterion for optimality of the outcome, then selecting optimal actions is trivial. *Decision theory* addresses the problem of acquiring information and making decisions in the presence of uncertainties with respect to the state of the world and the consequences of actions, when the optimality criterion is known (Pratt, Raiffa, & Schlaifer, 1964; North, 1968). As such, it is one approach to evaluating information value, in the Howard sense, for this class of problems.

*Decision analysis* provides a rational methodology for making decisions in the presence of uncertainty and for incorporating the worth of acquiring information to reduce uncertainty (Keeney & Raiffa, 1976). The approach is characterized by the decision maker enumerating all available decisions, quantifying subjective probabilities that the feasible states of the world will occur, determining the *values* (i.e., profit or cost) of all possible *outcomes* (i.e., combinations of decisions and states of the world), and assessing the *utility* function which describes the desirability of the values.<sup>2</sup> The concept of *utility* is employed to transform the *value* from an absolute scale (e.g., a monetary scale) to a scale that captures the individual decision maker's preferences. The decision objective is always to maximize the expected utility.

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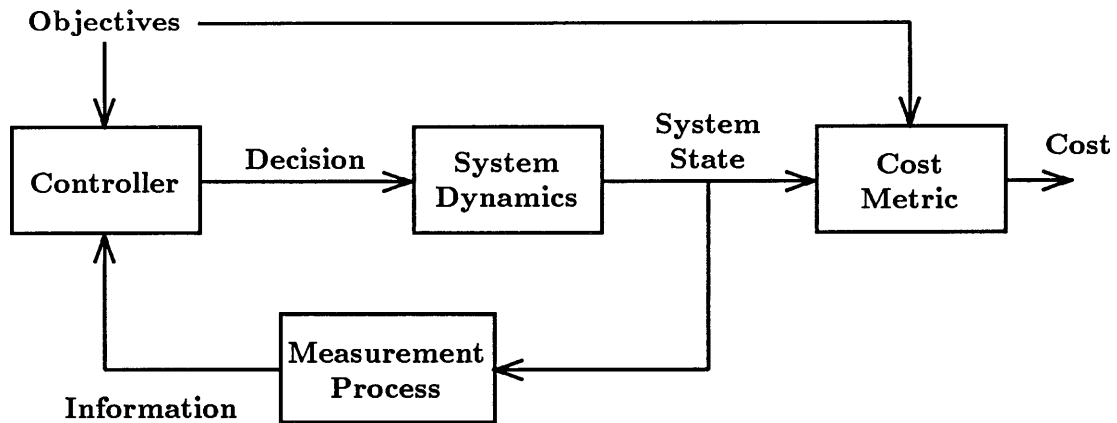
<sup>2</sup> The *outcome* of a decision  $a$  is the combination of the decision and the state of the world  $x$  which occurs,  $\{a, x\}$ . The *value*  $V(a, x)$  of the decision is an evaluation, measured on an absolute (e.g., monetary) scale, of the result of selecting action  $a$  in state  $x$ . The *utility*  $U(a, x)$  of an outcome  $\{a, x\}$  is an evaluation, measured relative to the individual decision maker's preferences, of the desirability of the outcome or the associated value  $V(a, x)$ . The *utility function* is, in general, a nonlinear mapping of the outcome or value into the utility.

By basing decision processes on a formal definition of rationality, decision analysis provides a tool through which assumptions, objectives, information sources, and feasible courses of action may be considered in a consistent manner (Raiffa & Schlaifer, 1961). The solution to a decision problem assumes a set of axioms that describe “rational” behavior and define the optimality criterion. The theory contends that the “axioms of rational behavior” are intuitively appealing and, therefore, all decision makers *should* act in a manner consistent with the axioms. A *normative* model, decision theory does not attempt to model how real decision makers behave, but rather describes how decision makers should behave to make analytically optimal decisions (Drake & Keeney, 1979). In contrast, *behavioral* models of human decision making, also known as descriptive models, are based on empirical evidence. The validity of assuming a maximum expected utility decision process has been demonstrated for a large class of problems in which the attributes of the utility measure are directly valued in economic terms. However, recent research (Patrick, 1996) suggests that maximizing the expected utility is not an appropriate model for how pilots make safety-critical decisions.

Decision analysis requires both a utility function  $U(a, x)$  that reflects the desirability to the decision maker of selecting action  $a$  when the state of the world is  $x$ , and a description of the decision maker’s a priori knowledge of the state of the world,  $P(x)$ . When this necessary data is available, decision analysis provides a methodology for choosing decisions that are optimal with respect to the decision maker’s preferences (Hiller & Lieberman, 1990). However, the theory does not provide a mechanism for measuring these necessary quantities. Although a large body of literature exists on methods for assessing subjective probabilities as well

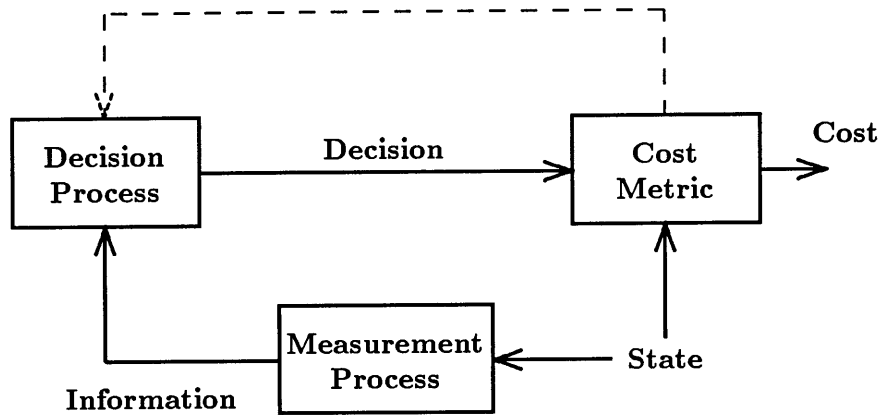
as single and multiple attribute utility functions, evidence suggests that decision making and valuing of information depends strongly on the specific decision and the individual decision maker.

The cost of acquiring information is, in general, directly measurable. In contrast, the effect which information will have on expected utility may be difficult to predict. Figures 2.1 and 2.2 show two typical architectures for decision problems. Figure 2.1 shows the classic structure of a feedback control problem. A cost function measures the state that is achieved through the control action, relative to the objectives. Figure 2.2 shows the open-loop structure used in decision theory. The cost function measures the cost for the combination of decision and state that occurs. In both classes of decision problems (open and closed-loop), the sensitivity of cost to information depends on both the cost metric and how the information affects the decision, typically neither of which are well known. A model of how the decision effects the state, in Figure 2.1, is typically available.



**Figure 2.1.** The relationship between information and cost in a feedback control problem.





**Figure 2.2.** The relationship between information and cost in open-loop decision theory.

Decision analysis represents a framework in which to model useful abstractions of complex, real decision problems, as they appear to individual decision makers. As a tool for selecting optimal decisions, the validity of the solution will strongly depend on the ability of the decision maker to capture important aspects of the decision problem in terms of the feasible actions, the uncertainties, and the utilities of possible consequences. As an approach to valuing information, decision theory depends on the validity of assuming a maximum expected utility decision policy and on the ability to model the decision maker's utility function and situation awareness. Furthermore, decision theory does not address the dynamic nature of the environment, the information, or the decision. Chapter 3 will introduce an approach to valuing information that accommodates the time-dependency which results from these dynamics and that is not as restrictive about assuming a known decision policy. Later chapters will study this model and the limitations of assumptions associated with a "normative" approach.

### 2.2.2 Information Value in Decision Theory

Decision theory defines the *expected value of perfect information* (EVPI) as the amount by which the expected value  $\alpha$  of the decision that would be made if the true state of the world were known (i.e., with knowledge of perfect information) exceeds the expected value  $\beta$  of the decision that would be made if no additional information were available. The EVPI is concerned only with the consequence of the terminal decision and does not consider the availability of intermediate information sources (Drake & Keeney, 1979).

$$EVPI = \alpha - \beta \quad (2.2a)$$

$$\alpha = \sum_x P(x) \max_a [V(a, x)] \quad (2.2b)$$

$$\beta = \max_a \left[ \sum_x P(x) V(a, x) \right] \quad (2.2c)$$

In (2.2),  $a$  is the decision,  $x$  is the state of the world,  $P(x)$  is the subjective, a priori probability distribution for states occurring, and  $V(a, x)$  is the value of selecting action  $a$  in state  $x$ . In (2.2b), the decision problem is to select the optimal action for the particular state of the world that has been realized. The expected value of the decision made with perfect information is still an expectation over the feasible states of the world; knowledge of perfect information does not change the likelihood of states occurring from the a priori probabilities. In (2.2c), the decision problem is to select a single action which will be applied regardless of which state of the world

occurs (i.e., the best decision that can be made when knowledge of the state of the world is described by the a priori probability distribution). Therefore, the role of perfect information is to reveal the state of the world before the decision is made, allowing a different decision to be made for each state as that state is realized. With respect to computing the expected value of a decision, information does not change the probabilities of states occurring from the original a priori estimates, only the knowledge on which the decision is based.

The expected value of perfect information may be generalized to allow arbitrary utility functions, in which case the value equals the purchase price for perfect information such that the decision maker is indifferent between making the terminal decision with or without acquiring the information (Pratt, Raiffa, & Schlaifer, 1964).

Because the available sources of information may not fully reveal the state of the world, the expected value of perfect information defines a bound on the amount which a decision maker would be willing to pay to receive information. The a priori probabilities, based on the initially available information, represent the decision maker's subjective perception of the likelihoods that each feasible state of the world will occur. Additional information conditionally modifies these probabilities according to Bayes' rule (2.3).

$$P(x | I) = \frac{P(I | x) P(x)}{P(I)} \quad (2.3a)$$

$$\text{where } P(I) = \sum_x P(I | x) P(x) \quad (2.3b)$$

$P(I | x)$ , the probability of the received information being  $I$  if the true state of the

world is  $x$ , must be known a priori from the problem statement. Given information  $I$ , the description of the decision maker's knowledge about the state of nature becomes  $P(x | I)$  and, in contrast to (2.2c), the expected value of the decision with new information equals:

$$\max_a \left[ \sum_x P(x | I) V(a, x) \right]. \quad (2.4)$$

The value  $V_I$  of information  $I$ , therefore, equals the difference between the expected values given in (2.4) and (2.2c).

$$V_I = \max_a \left[ \sum_x P(x | I) V(a, x) \right] - \max_a \left[ \sum_x P(x) V(a, x) \right] \quad (2.5)$$

Information value will be positive unless the information misleads the decision maker toward an action that is less appropriate for the true state of the world. Notice from (2.2) or (2.5) that the definition of information value assumes a model for the decision process – choosing the action that maximizes expected value. Furthermore, the chosen optimal actions in the two decision problems, with and without information, are allowed to be different. Also notice that, through the value function  $V(a, x)$  and the initial knowledge  $P(x)$ , decision theory values information in the context of the way in which the information is applied.

Prior to the content of the information being revealed, *expected* information value,  $E(V_I)$ , is given by the expectation over the possible pieces of information.

$$\begin{aligned} E(V_I) &= \sum_I P(I) \max_a \left[ \sum_x P(x | I) V(a, x) \right] \\ &\quad - \max_a \left[ \sum_x P(x) V(a, x) \right] \end{aligned} \quad (2.6)$$

If the information completely reveals the true state of the world (i.e.,  $P(x) = 1$  when  $x$  equals the true state of the world and 0 otherwise), (2.5) may be rewritten as (2.7). This expression for the value of perfect information depends on the state of nature and, accordingly, differs from the expression for *expected value of perfect information* (2.2) because the probabilities of states occurring are different.

$$V_I = \max_a [V(a, x)]_x - \max_a \left[ \sum_x P(x) V(a, x) \right] \quad (2.7)$$

## 2.3 Modern Literature

### 2.3.1 Relating Shannon Information to Howard Information Value

Sheridan (1995) proposed a general framework for analyzing information management decisions based on measuring and valuing information. Reflecting on the relation between the classical theories of information and information value, Sheridan concluded that the two concepts are independent, characterizing different aspects of information seeking and using, but complementary. Although the Shannon measure of information quantifies the effort or cost required to discover the truth from an initial state of uncertainty, it does not consider the benefit realizable by applying that knowledge. Similarly, information value as defined by Howard measures the increased profit that is achieved by taking action with reduced uncertainty, but excludes the effort or cost of acquiring the information necessary to reduce the uncertainty. Sheridan recognized that if information is scaled to the units of information value (e.g., by the cost per bit), then the difference between information value

and scaled information would represent the tradeoff between the benefit of having information and the cost of acquiring information. Offering this model to study information seeking/using behavior in decision-making processes, Sheridan suggested that information should be provided when the benefit of knowing the information, which Howard and decision theory measures as the increase in the expected utility of the decision outcome, exceeds the cost of providing the information.

However, since the existing theories which Sheridan's model exploits do not capture the inherent time-dependence of measured information in a dynamic environment, the model does not address the issue of when to update temporal information. Nor does the model accommodate time-dependence in the decision.

### **2.3.2 Models of Information-seeking Behavior**

Sheridan has studied several issues relevant to this thesis. Sheridan (1992) considered the attentional demands on a supervisor responsible for monitoring several displays. Under the simplifying assumptions that there is no cost for taking observations and the observation times and time to transition between displays are negligible, the supervisor should sample each display often enough to capture the highest frequency component present in that display. The minimum sampling frequency – the Nyquist frequency – is two times the display's bandwidth. The second assumption implies that workload considerations do not constrain the solution. However, in general, the supervisor has a finite amount of time available for making observations, which must be allocated between multiple displays and between sample resolution or accuracy (i.e., the dwell time on a display for a single observation) and sample frequency.

Senders et al. (1964) confirmed experimentally that experienced subjects confronted with multiple displays attend to each display with frequency directly proportion to the display's bandwidth. Senders et al. observed that subjects tended to over-sample low frequency displays, possibly because the subjects forgot the content of the display before one over the Nyquist frequency after the previous sample, and under-sample high frequency displays, possibly because the observations were not instantaneous, allowing subjects to perceive the display's velocity.

The conclusion that the most efficient observation interval is one over the Nyquist frequency assumes that the signal being displayed has finite bandwidth. If the bandwidth of the signal changes at an unknown time (e.g., the signal remains constant for a long time and then exhibits a step-change), the above model provides little insight into how an observer will or should sample that signal. The observer's task is to monitor a low frequency signal so as to detect an occasional high frequency event (e.g., a step-change in an otherwise constant signal). Since the human observer does not know when to sample the display to detect the infrequent event, he must sample at a rate that is sufficiently high such that the delay in detecting the event does not exceed the acceptable response time. Alternatively, an automatic alerting system can sample the display at a very high rate and use this knowledge of the signal to determine when to either force the human to make an observation of the display or announce the current value of the display. This situation is identical to that in cooperative versus non-cooperative information management environments, in which the "alerting system" can use knowledge of the information to measure the value an "alert" would have to the "observer" (Section 4.4).

Sheridan (1970) extended the analysis of when a supervisor should seek in-

formation by considering a problem in which the supervisor must decide when and how to adjust a set of controls and when to acquire information to support those decisions. Sheridan assumed that the supervisor's objective was to maximize the expected return from the decision, which was a known measure of the controls and the state of the world. Costs for acquiring information and adjusting the controls were included.

Sheridan recognized that the supervisor's knowledge about a display degrades as the time since the last observation of the display increases. In general, observations made prior to the most recent observation also contribute to the supervisor's current knowledge about a display, because the supervisor may use the sequence of observations to predict the future content of the display. Sheridan modeled the supervisor's knowledge about the display as converging to some statistical expectation, independent of the observation, as the age of the observation increased. The observation was modeled as a sample taken from this stationary distribution. In general, the rate at which the variance of the supervisor's model of the display increases (until it reaches the variance of the a priori distribution for the display) depends on the bandwidth of the signal underlying the display and the number of derivatives measured at each observation.

Sheridan used this problem formulation to consider several cases of constraints on sampling and control adjustments. Without any sampling (e.g., if the costs of information or control adjustments are high relative to the return from the decision), the optimal strategy is to adjust the control once, to maximize the value assuming the only knowledge about the state is the statistical distribution for the display. Alternatively, if the information and control costs are zero, the supervisor could



sample the display and adjust the control continuously, to maximize the value for each particular observation encountered. Between these extreme cases, a supervisor may sample intermittently but continuously adjust the control based on his estimate of the state between the observations. The maximum expected values for this result, in the limits as the sample interval is made very small and very large, equal those from the previous two cases.

Sheridan also considered the case in which the supervisor samples intermittently (at a constant interval) and the control may only be adjusted at the times of the observations. Sheridan shows that the selection of the optimal sampling interval is a tradeoff between the high cost per unit time of acting (i.e., sampling and adjusting the control) frequently and the low return per unit time of acting infrequently. More generally, the supervisor may choose not to adjust the control at every sample, if the improvement in the return would be less than the cost for adjusting the control. Sheridan also generalized the expressions for the maximum expected value to accommodate sensor errors in the measurements as well as the case in which the relevant states are not directly observable, requiring estimates to be constructed from measurements of observable states.

Sheridan acknowledges that determining the value function for a particular decision problem, on which his results are predicated, is the “most difficult” problem in analyzing human decision-making. Moreover, the assumption that decisions are made to maximize the expected value underlies all of these models which attempt to describe information-seeking behavior in various situations. Sheridan (1992) concluded by recognizing that this assumption is likely to be inappropriate for many decision makers, such as one who is risk-averse.

Finally, the supervisor's capabilities constrain the amount of information that may be collected or how often each control may be adjusted. If the attentional demands specified by the solution to the previous problem exceed the supervisor's capabilities, the problem becomes that of allocating the supervisor's resources (sensory, cognitive, and motor) between the multiple responsibilities. One approach is to model these constraints by assessing costs for using the supervisor's limited resources, and apply the previous methodology. Sheridan (1992) discusses dynamic programming as an approach to optimally allocating a supervisor's resources. In a bandwidth-limited communication system, the available bandwidth must be shared between multiple decisions, limiting the information that may be provided to support each decision. An approach similar to Sheridan's might address aspects of this problem. In general, the problem of allocating limited resources to achieve multiple objectives has received significant attention in the literature.

### **2.3.3 Alternative Information Value Theories**

Following the approach taken by Howard, Copper (1992) also recognized that the classic (i.e., Shannon) definition for information quantity is based on the unexpectedness of the message and omits a measure of the significance of the message. That not all messages which are equally unexpected are equally meaningful suggests the importance of the state of the information user, as well as the context in which the information is applied, in determining the impact of the message. Wagner (1990) supported this perspective, commenting that the concept of information value is subjective, depending on the user of the information, the context in which

the information is applied, and time. Similarly, Taylor (1982) stated that information value may only be defined in the context of its usefulness to an information user.

Copper distinguished between the importance assigned to a message by the information user, which is subjective in nature, and the information utility, which he defined, in the context of an information user and the goal which the user endeavors to attain, as an objective measure of the necessity and sufficiency of the message for the user to achieve the goal. Copper notes that this perspective allows the utility definition to reflect the fact that the message was received and how the user will benefit, rather than the state of the world implied by the message and the optimal benefit that could be achieved as a result of the world being in that state. Copper concluded that automation could fully appreciate the importance of information only after being programmed with “artificial empathy” – knowledge of the goals of the user and their relative importance and the ability to judge the impact of the information on each goal. Similarly, Morehead and Rouse (1985), conducting empirical experiments in a database search environment, observed that the searchers’ value structures were apparent in the information seeking process, and concluded that understanding how humans value information is an important consideration in designing interactive information systems.

In the context of applying multiple observations of the same quantity or state, separated in time, to reduce a single source of uncertainty (i.e., improve the estimate of that state), Charnetski and Conerly (1986) studied the value of new measurements which decrease the age of the observations on which the estimate is based. The sample problem was to predict future outcomes of a Polya random process.

They observed that obtaining more current data for a Polya-type process, which is statistically stationary, has positive value (i.e., improves the prediction) only under certain conditions, and, in such cases, the value tends toward zero as the number of trials of the process increases (i.e., additional information has diminishing value). From these results, Charnetski and Conerly concluded that reducing observation delays in certain related processes may not effect significant improvements in decision making. Furthermore, the value of decreasing data age may not justify the cost of collecting more current data.

In the context of applying multiple sources of information to reduce a single source of uncertainty, Heckerman et al. (1993) studied the optimality of the sequence in which the observations are made. Heckerman et al. proposed that decision makers use information value as a criteria both to decide whether or not to collect additional information before selecting a final action and, if additional information is sought, to decide what observation to make next. In general, after learning the results of previous observations, and given the option of making several additional observations, determining which observation will provide the maximum information value if made next, requires considering the potential long-term values of making all possible sequences of the feasible observations. Exhaustive analysis of the order in which observations should be made is typically intractable because the number of sequences grows exponentially with the number of feasible observations. A common approach to avoid the intractability of an exact information value computation involves the *myopic* assumption – that only one additional observation will be made before an action is chosen by the decision maker. Heckerman et al. observed that this assumption may sacrifice optimality in both the choice of observations and the

choice of the final action. Heckerman et al. introduced an approximate non-myopic method for computing the value of information for a series of observations that exploits statistical properties of large samples. Although not as general as an analysis in which all possible sequences of observations are considered, the approximation is linear, rather than exponential, in the number of feasible observations.

The majority of research concerning optimal strategies for seeking and using information has focused on single and multiple sources of information in the context of a single source of uncertainty. Samson et al. (1989) considered problems in which the decision maker chooses a set of information to reduce multiple sources of uncertainty. In general, the expected values of perfect information, in the context of multiple sources of uncertainty, are not additive. Samson et al. demonstrated that making acquisition decisions about multiple sources of information to addresses multiple uncertain quantities in isolation of each other is generally sub-optimal. In the case of quadratic decision problems, Merkhofer (1977) noted that if two pieces of information are uncorrelated, then the value of obtaining both pieces simultaneously approximately equals the sum of the values of obtaining the two pieces independently. Samson et al. developed general necessary as well as a necessary and sufficient conditions for additivity of expected values of perfect information. In addition, Samson et al. discussed the implications of multiple uncertainties on the expected value of imperfect information, as well as on the expected utility of information, which incorporates a nonlinear risk function.

### 2.3.4 Dynamic Decision Theory

The *static* decision theory which has been discussed is founded in the subjectively expected utility maximization model – the decision maker is confronted by a well-defined set of possible actions where, for every feasible state of the world, values are associated with selecting each action and subjective probabilities describe the likelihoods of the states being realized. Static decision problems end after the decision maker selects the action which maximizes his personal utility for the expected outcome (Edwards, 1962). Moreover, in static decision problems, the state of the world is time-invariant.

*Dynamic* decision theory was introduced by Edwards (1962) and Toda (1962) in studying human decision-making. In dynamic decision theory the environment may change in time, either as a function of or independent of earlier decisions, while the decision maker collects information about the environment. Edwards notes that a mathematical treatment for decision problems involving nonstationary environments is often unavailable. The environment is *stationary* when it may be described by a stochastic process with constant statistics.

Rapoport (1975) and Brehmer (1992) suggest that dynamic decision problems are characterized by four properties: a series of decisions, each yielding payoff and information, is required to achieve a goal; the decisions are dependent (i.e., later opportunities are constrained by earlier decisions); the decisions must be made in real-time; and the state of the world changes both autonomously and as a consequence of the decision maker's actions.

Static decision theory represents a normative approach to studying human decision making – a mathematically optimal course of action is proposed and the

degree to which human decision makers deviate from the model is studied. Brehmer (1992) observed that the classic normative theory of decision making is not useful for studying dynamic decision problems. Rapoport (1975), following the approach taken by Edwards (1962), introduced a hypothetical, ideal decision maker and compared the behavior of real decision makers. Although Rapoport provided solutions for several example sequential decision problems, he concluded that the normative approach is limited by the inability to analytically determine optimal solutions to many dynamic decision problems. Similarly, Toda (1962) introduced a paradigm for studying dynamic decision making for which analytic solutions were available for only simple versions. Subsequent research studying human decision making in dynamic environments has largely abandoned normative approaches in favor of behavioral models based on empirical studies (Rapoport, 1975).

Edwards (1962) and Neimark (1961) conducted experiments on human information seeking behavior and decision making. In each experiment, subjects would make a decision after purchasing information that modified their opinions concerning either probabilities or payoffs. Although individuals were self-consistent, all three studies demonstrated significant differences between subjects, with a common tendency to seek excessive information. Furthermore, the subjects' strategies were shown to be sensitive to costs, payoffs, and probabilities. Moreover, all three experiments demonstrated that human decision makers do not act to maximize expected value (Edwards, 1962). Edwards concluded that the size and ubiquity of the differences between subjects in these experiments is discouraging for developing a general model for human decision making.

*Intuitive statistics* relates a decision task in an environment where the infor-

mation is changing to a problem in statistical estimation. Irwin et al. (1956) and Irwin and Smith (1956; 1957) studied the ability of humans to intuitively estimate simple statistics from unknown random processes. Subjects were shown samples from a random process and asked to estimate the mean and variance as well as indicate their confidence in their estimated mean. The studies showed that humans are capable of estimating average values on an intuitive basis, but do not intuitively perform other statistical tasks (e.g., estimating variance) well. The confidence ratings for the estimated means increased with sample size and decreased with process variance. A similar study allowed Edwards (1962) to conclude that humans function rather well as Bayesian information processors, in stationary environments, combining new and old information to learn probabilities.



## Chapter 3

# A Theory of Time-Dependence in Information Value

The previous chapter reviewed much of the progress that has been made both in general theories of information value and in applying those concepts to the problem of managing the measurement and communication of information. The following chapters extend these existing results by studying more closely the time-dependence of information value and its implication for information management techniques. Several questions are asked. How does the age of information, which describes the condition of a state variable at a prior point in time, affect its ability to support the making of a decision at the present time? If one or more new measurements may be taken, how does the value of the information represented by those measurements vary with the time at which the measurements are taken? When – how frequently or under what conditions – should measurements be taken to support a decision-making process?

To address these questions, this chapter develops a novel model for describing the time-dependence of information value. The approach builds on the definition of

information value established in decision theory and applies estimation techniques to model the effect of time.

### 3.1 Proceduralized Decision Problems

Notice from the above questions that the goal of information management, in this analysis, is to support decision-making, by efficiently controlling the measurement and communication of information. Therefore, information is valued in the context of the decision to which that information is being applied.

This thesis considers *proceduralized decision problems*. Prevalent in aviation, proceduralized decision problems are a class of decision problems in which an established procedure or rule specifies the correct decision or action as a function of one or more *relevant* states of the world. Thus, we investigate information value in the context of decision problems with fixed decision rules, as opposed to the broader class of decision problems (e.g., automatic control) in which the decision rules optimize an explicitly stated criteria. Patrick (1996) observed that “most decisions made in the cockpit are related to safety, and have therefore been proceduralized in order to reduce risk.”

Decision problems for which a pilot is required to obey a Federal Aviation Administration (FAA) regulation or airline operating procedure belong to this class. For example, the airport surface conditions for which a category I ILS (Instrument Landing System) landing is permitted are defined by FAA regulations. Therefore, whether a pilot is allowed to land (or is required to perform a missed-approach)

rigidly depends on the *relevant* state variables – the airport surface conditions (e.g., the runway visual range (RVR) at the time the approach is flown). However, prior to arriving at the airport, whether the pilot decides to proceed or divert to an alternate airport depends on the pilot’s estimate of what the airport surface conditions will be upon arrival, as well as elements of the context of the decision such as available fuel reserve and the inconvenience of landing at an alternate airport.

Therefore, the decision maker’s task in a proceduralized decision problem is to estimate the relevant state variables (or predict what the relevant state variables will be at a future time of interest, e.g., the time at which the aircraft would arrive at the primary airport). This thesis assumes that, if the state variables are known accurately, all decision makers will choose the action called for by the procedure. Note that a bad procedure may call for a decision that does not minimize the cost which results from the decision.

Pilot tasks such as monitoring aircraft altitude or engine temperature may also be viewed as proceduralized decisions. Monitoring decision problems are characterized by the decision maker continuously choosing either to delay taking an intervening action while continuing to monitor or to take an intervening action. This decision whether or not to intervene depends on the condition of a state variable relative to a criterion specified by the procedure. For example, if the engine temperature deviates by more than an allowed tolerance from a normal operating point, the pilot is required to take an action. Otherwise, the pilot continues to monitor.

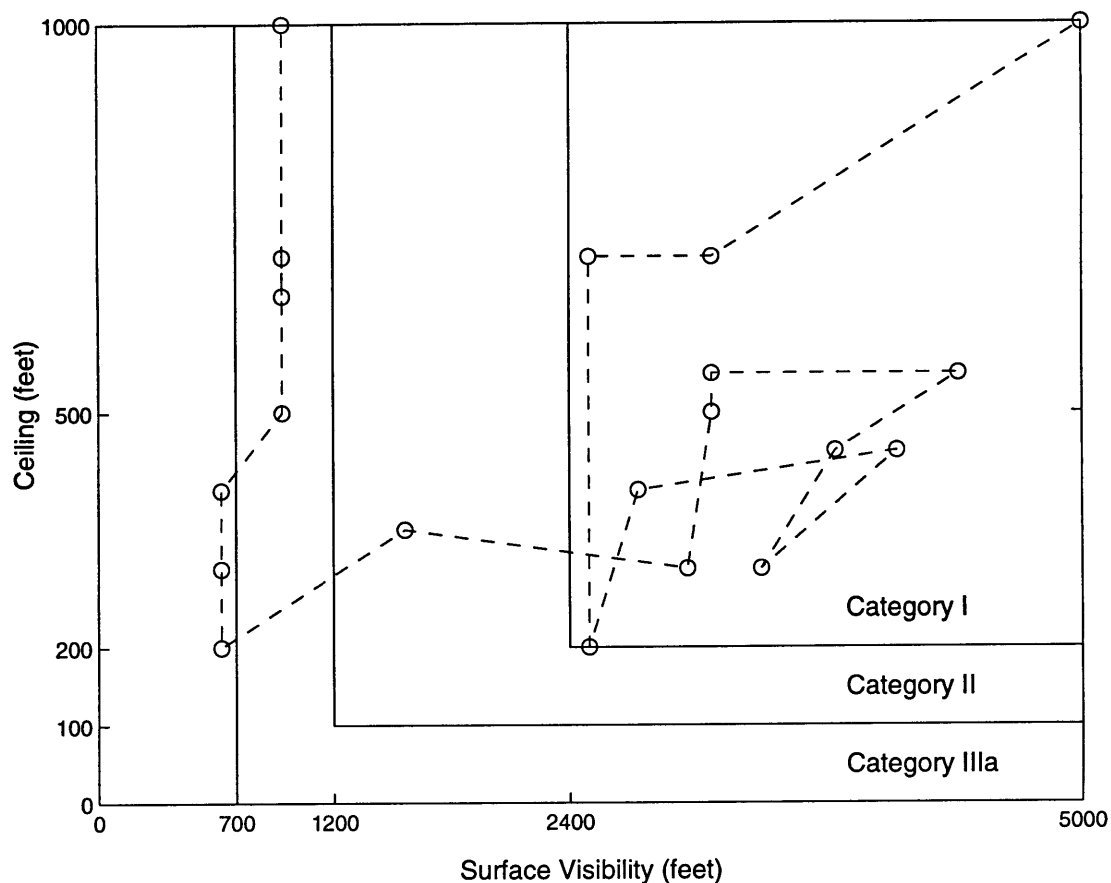
### 3.1.1 Threshold Surfaces

*Threshold Surfaces* model the way in which a proceduralized decision problem depends on a set of  $S$  relevant state variables  $\mathbf{x} = [x_1, x_2, \dots, x_S]^T$ . The procedure is modeled by determining, for each point in the state space, which of a set of possible actions  $\{a_1, a_2, \dots, a_M\}$  is correct.  $M$ , the number of actions between which the decision maker must choose, is typically small and, therefore, each of the possible actions is appropriate for a set of points in the state space. The points in the state space for which the action  $a_i$  is called for by the procedure comprise the *region*  $r_i$ . By definition, the  $M$  regions are mutually exclusive and collectively exhaustive. Threshold surfaces are the boundaries around these regions. The symbol  $\theta$  is used to represent a threshold in a single-dimensional state space. Note that the correct action does not depend on where the state vector lies within a region. Therefore, the decision maker's task in making a proceduralized decision is to identify the region in which the state lies (or will lie at a future time of interest).

For example, the procedure for a precision instrument approach requires that a missed approach be flown if, when the aircraft reaches the Missed Approach Point (MAP) by following the glide-slope, the ceiling is less than the Decision Height (DH) or the surface visibility along the runway is less than the Runway Visual Range (RVR), such that the pilot cannot complete the landing using visual guidance.<sup>1</sup> The Decision Height and Runway Visual Range are thresholds in the proceduralized decision problem – whether or not to perform a missed approach. The DH and RVR are determined by the Category (CAT) of the instrument approach, and

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<sup>1</sup> The Missed Approach Point is defined as the point on the glide-slope that is the Decision Height above the ground (Simpson, 1993).



**Figure 3.1. Decision Height and Runway Visual Range requirements define regions for which Category I, II, and IIIa ILS landings are permitted.** To illustrate the dynamic nature of airport surface conditions, representative ceiling and surface visibility data is plotted at 5 minute intervals.

the Category (e.g., III, II, I, or non-precision) is determined by the minimum of the certification of the runway and the aircraft equipment. Figure 3.1 shows the lowest authorized DH and RVR requirements for CAT I, II, and IIIa approaches, using Instrument Landing System (ILS) navigation equipment, as thresholds in the ceiling/surface visibility state space (Aeronautical Information Manual, 1996). Requirements at a particular runway may be higher when necessitated by charac-

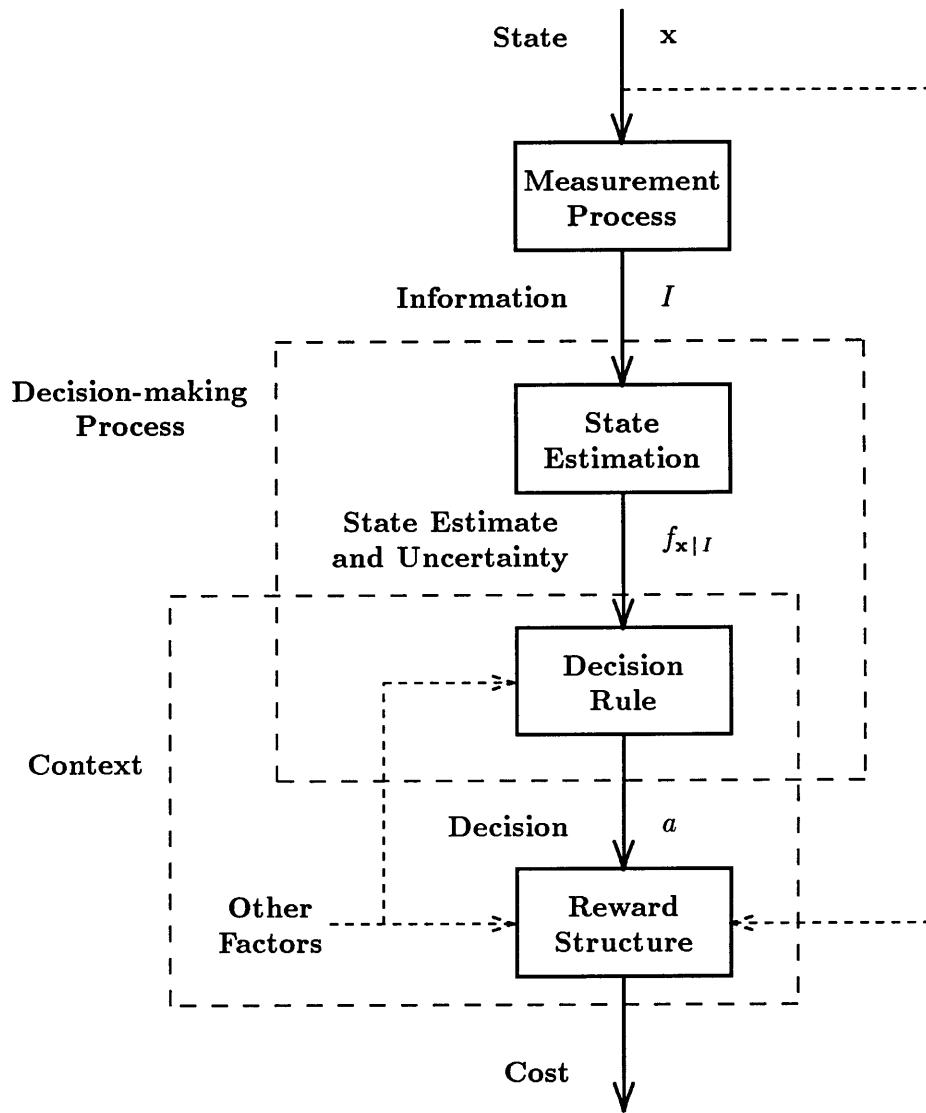
teristics of that site, typically along the Missed Approach path. For example, the Decision Height and Runway Visual Range requirements for a CAT I ILS approach to Runway 22L at Boston's Logan International Airport are 404 feet and 6000 feet, respectively.

The DH and RVR are requirements on the instantaneous surface conditions at a specific point (i.e., at the point on the glide slope that is the decision height above the ground, looking along the runway). Airport surface observations are the principle information on which a pilot must decide whether to attempt a landing or divert to an alternate airport. Figure 3.1 also plots ceiling and surface visibility data representative of the conditions which a pilot might encounter. In general, thresholds and regions may also vary with time, for example if they define an area of the state space which contains a time-varying hazard that must be avoided, such as convective weather or another aircraft.

### 3.1.2 Separation of Estimation and Context

The previous section showed that the solution of a proceduralized decision problem is found by solving an estimation problem (i.e., by identifying the region of the state space in which the state vector lies). To measure the value of new information to the decision problem requires first understanding how the new information will affect the solution to the estimation problem and, second, how that change in the knowledge about the state will affect the decision outcome. This section shows how these two steps will be considered separately.

Figure 3.2 is a model of the steps in the solution of a proceduralized decision problem. First, the state  $\mathbf{x}$  takes on a certain value, which is measured, producing



**Figure 3.2. Separation of the decision-making process into cascaded state estimation and decision selection.** A filter constructs a model  $f_{\mathbf{x}|I}$  of the state variables  $\mathbf{x}$  from the information  $I$ , and a decision rule defines an action  $a$  based on the model of the states.

information  $I$ . The *decision-making process*, selecting an action  $a$  given the information  $I$ , will be separated into two cascaded steps: constructing a model  $f_{\mathbf{x}|I}$  of the relevant states from the information, and using that model to select an action  $a$ . Modeling the knowledge about the state with a probability density function  $f_{\mathbf{x}|I}$  allows the uncertainty as well as the state estimate to be used in selecting the decision. This separation parallels the separation theorem of estimation and control, in which an optimal controller may be constructed by concatenating an optimal estimator and an independently designed optimal full-state feedback control law. Also note that Shannon's (1949) theory of information is concerned with the sensitivity of the state model  $f_{\mathbf{x}|I}$  to the information  $I$ , while Howard's (1966) information value theory is concerned with the sensitivity of the cost resulting from the decision to the uncertainty in the state model.

For the class of proceduralized decision problems, the value which a piece of information has to the decision-making agent is defined as the change which knowledge of the information effects in the probability that the decision-making agent will choose a sub-optimal decision, weighted by the amount by which the cost of the sub-optimal decision exceeds the cost of the optimal (i.e., minimum cost) decision. The decision-making agent may choose a sub-optimal decision due to uncertainty in the model of the state variables on which the procedure depends. Information improves the decision-making agent's model of these state variables.

This definition is subtly different from the classic *expected value* definition, explained in Chapter 2, which measures the effect of the information directly on the cost of the decision. By measuring the sensitivity of the cost  $C$  to the information  $I$ , the classic definition obscures the estimation process, which is central to



proceduralized decision making. In the class of proceduralized decision problems, the goal of providing information is to improve the decision-making agent's knowledge about the relevant state variables. The motivation for introducing a different information value metric is to more directly measure the impact of information on this goal. The information value metric developed in the following section measures the effect of the information on the ability to model the relevant state variables (i.e., the sensitivity of the model  $f_{\mathbf{x}|I}$  to the information  $I$ ), in the *context* of the proceduralized decision problem.

The *context* within which the information is valued consists of the decision rule (i.e., how the decision maker selects the action  $a$  given the model  $f_{\mathbf{x}|I}$ ) and the reward structure (i.e., the costs for every combination of the possible actions and the values which the state can take on). *Other factors* are aspects of the decision environment that will affect the decision but are not explicitly considered by the procedure.

Since the decision-making agent is constrained to operate within the procedure, if the procedure is bad (i.e., non-optimal in a minimum cost sense), classic information value theory may assign negative value to information that reveals the true state. The current approach will value that information in the context of the bad procedure (i.e., how the information helps the decision-making agent make the decision prescribed by the procedure).

Section 3.2 will introduce a general model of the decision-making agent's knowledge about a set of state variables, in relation to the threshold surfaces. Section 3.3 will extend this model to describe the effects of time and new information on the knowledge about the state variables. Section 3.4 will introduce a general

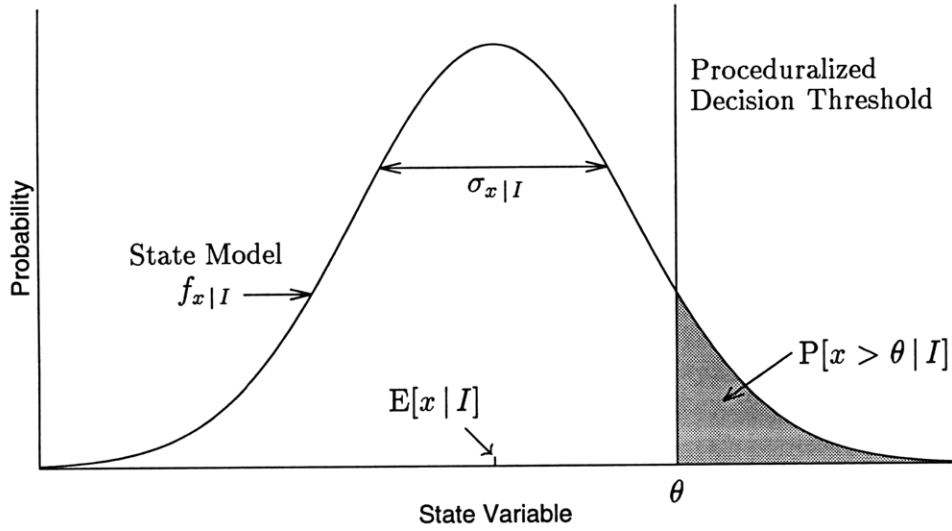
model for describing the *context* within which information is valued. Finally, Section 3.5 will apply these ideas to define the measure of information value that will be used through the remainder of the thesis.

### 3.2 Uncertainty and Uncertainty Significance

Knowledge about a set of relevant state variables  $\mathbf{x}$ , given the available information is  $I$ , is modeled by a conditional joint probability density function (PDF)  $f_{\mathbf{x}|I}$ , describing the relative probabilities that the state will take on each of the feasible values. Figure 3.3 shows a model  $f_{x|I}$  for a single state variable  $x$ , in relation to a threshold  $\theta$ . There is nothing special about the shape of the PDF shown in the figure. For example, if the only knowledge about the state is that it lies between a minimum and maximum value, then the density would be a constant across that feasible range. The threshold defines two *regions*:  $x < \theta$  and  $x > \theta$ . The probabilities that the state lies in these regions,  $P[x < \theta | I]$  and  $P[x > \theta | I]$ , respectively, constitute the probability mass function for the region in which the state lies,  $p_r$ . Table 3.1 summarizes the terminology used in describing the state model.

The term *state model* is used throughout this thesis to reference a description, typically a probability density function, of the knowledge about a set of state variables. The term *state dynamics model* will be used to reference a description of the underlying random process that drives the state variables.

A variety of established analytic approaches exist for identifying an estimate  $\hat{x}$  from the state model  $f_{x|I}$ . For example, the conditional mean estimate (which



**Figure 3.3.** A probability density function model for a state variable.

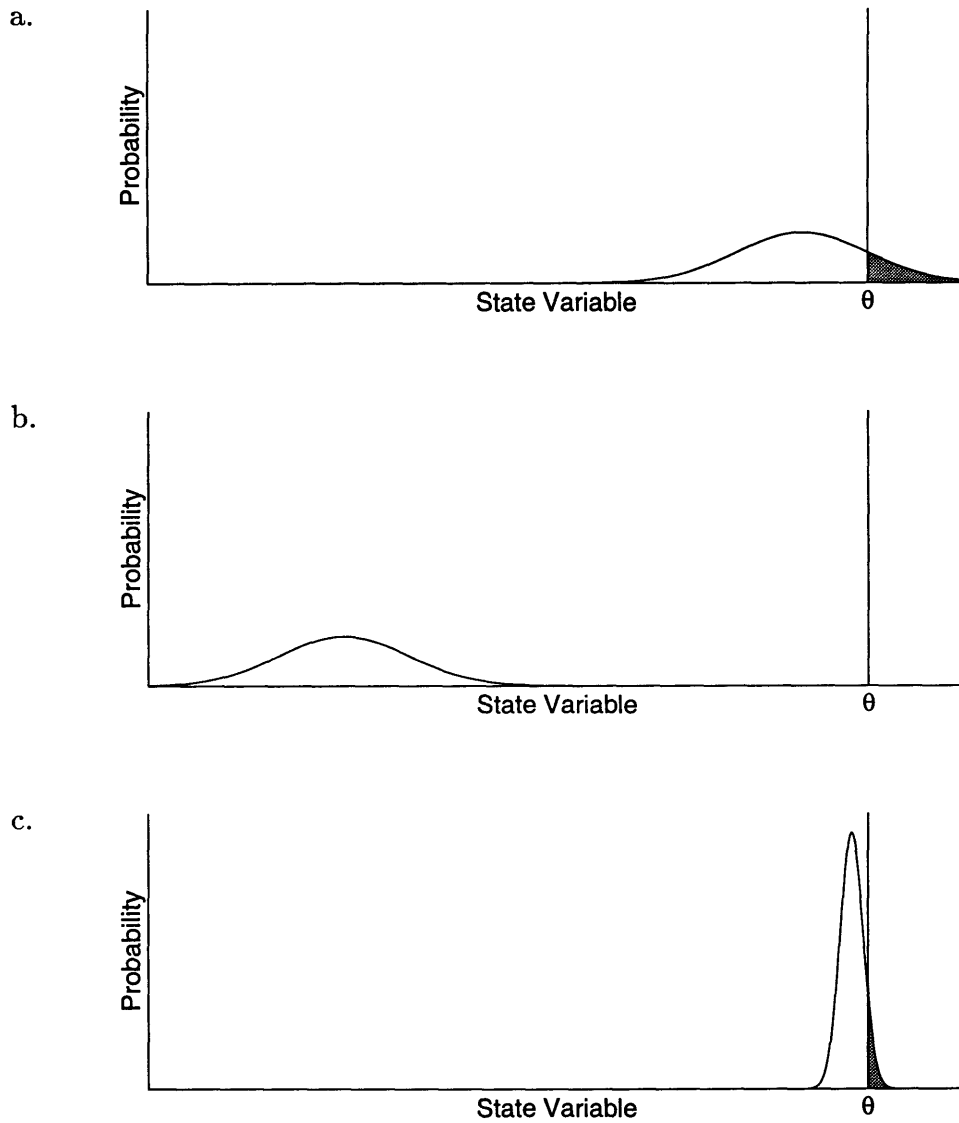
equals the minimum variance Bayes' estimate when  $f_{x|I}$  is Gaussian) is the expected value of  $f_{x|I}$ ,  $E[x | I]$ . Alternatively, choosing the estimate to be the mode of  $f_{x|I}$  maximizes the probability that the estimate equals the true value of  $x$  (Gelb, 1974). Individual decision makers would likely each have different internal models (PDF's) for the state and may use altogether different approaches for choosing an estimate.

The *error*  $\tilde{x}$  in the estimate of the state is the difference between the estimate and the actual state. However, to measure the error requires knowing the actual state, in addition to the estimate. When the error cannot be measured, the quality of the state estimate is often measured by the *uncertainty*. The *uncertainty* in the state model is defined as the *expected error* (i.e., the standard deviation) of the PDF.

**Table 3.1. State model terminology.**

Symbol	Definition
$\mathbf{x}$	Vector of relevant state variables
$\hat{\mathbf{x}}$	State estimate
$E[\mathbf{x}]$	Expected state
$f_{\mathbf{x} I}$	Model (probability density function) for the state $\mathbf{x}$ , based on information $I$
$r_i$	A region in the state space
$n$	Number of regions in the state space
$P[\mathbf{x} \in r_i   I]$	Probabiltiy that the state $\mathbf{x}$ is in region $r_i$ , based on the model $f_{\mathbf{x} I}$
$p_{r I}$	Probability mass function for the region in which the state will lie, based on the model $f_{\mathbf{x} I}$
$\theta$	Threshold surface

The state model having a large uncertainty, or a large error, will not necessarily affect the outcome of a proceduralized decision. For example, when the ceiling at an airport is expected to be 4000 feet, a  $\pm 1000$  foot uncertainty is not significant to a pilot planning to make a non-precision instrument landing, because the pilot is confident that the ceiling will be greater than the Minimum Descent Altitude (MDA). However, if the expected ceiling is within 50 feet of the Minimum Descent Altitude, a  $\pm 50$  foot uncertainty would be significant to the pilot's decision whether or not to divert to an alternate airport because he may not be able to land at the primary airport. Therefore, rather than considering the magnitude of the uncertainty (i.e., the standard deviation), a measure of how significant that uncertainty is in the context of the decision problem is required. Figure 3.4 reinforces



**Figure 3.4. The difference between the magnitude and the significance of uncertainty.**

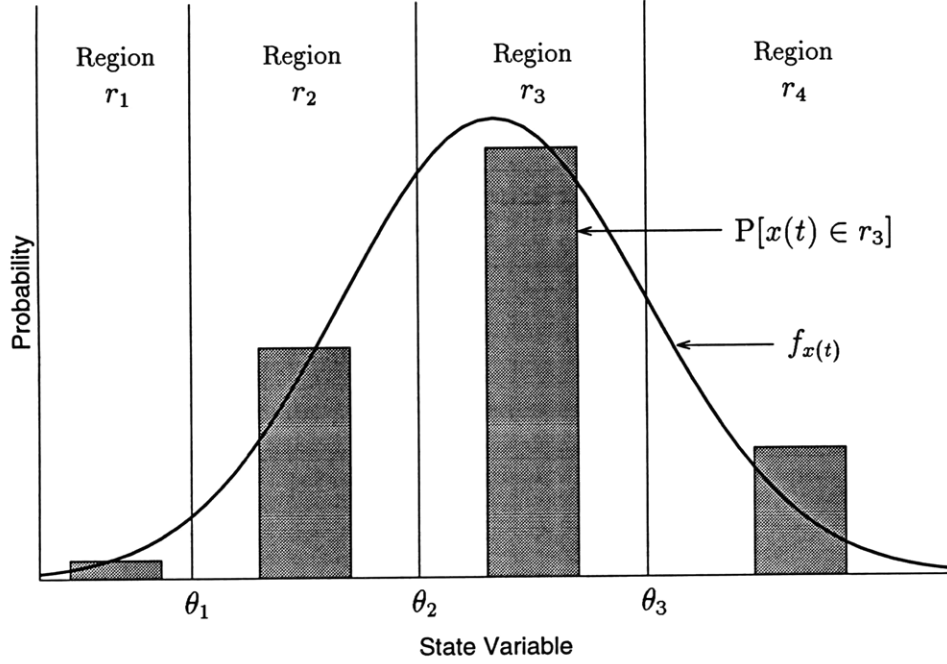
Although the standard deviations of the distributions in parts *a* and *b* are equal, in *b* there is no uncertainty concerning whether the state is greater or less than the threshold  $\theta$ .

Although the standard deviation of the distribution in *c* is smaller than that in *a*, the two examples exhibit similar uncertainty concerning whether the state is greater or less than the threshold.

this point. The two comparisons made in the figure, part *a* versus part *b* and part *a* versus part *c*, illustrate that the significance of uncertainty in a state model to a proceduralized decision depends on both the magnitude of the uncertainty and the proximity of the expected value of the state to the threshold.

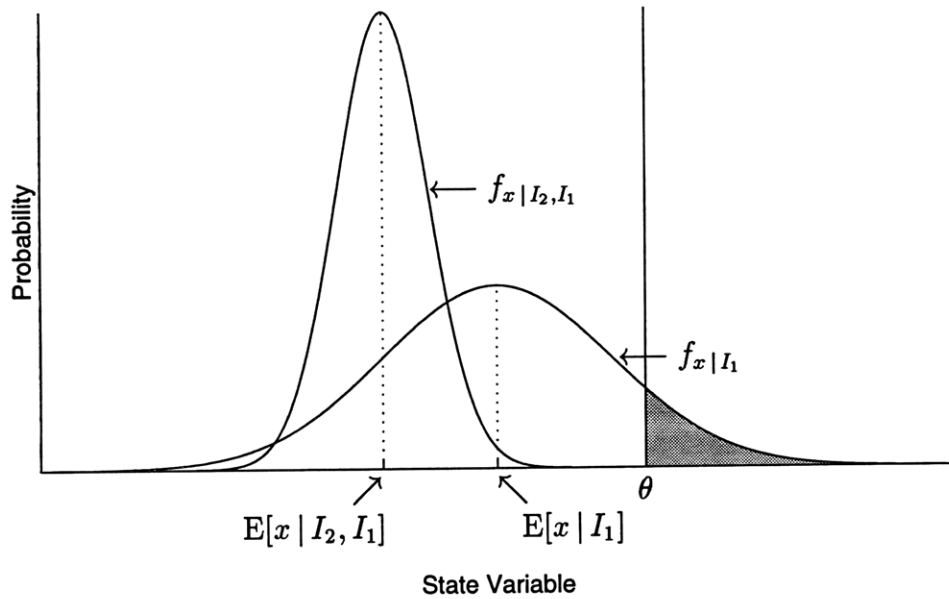
Uncertainty in the state model (i.e., the expected error of  $f_{\mathbf{x}}$ ) is significant to the outcome of a proceduralized decision when the region in which the state lies is not known. In a proceduralized decision problem, the decision maker's task is to correctly identify the region in which the state lies. The probability that the state  $\mathbf{x}$  is in region  $r_i$ , denoted  $P[\mathbf{x} \in r_i]$ , equals the integral under the probability density function  $f_{\mathbf{x}}$ , over all  $\mathbf{x}$  in the region  $r_i$ . Figure 3.5 shows a possible probability mass function  $p_r$  for the region in which the state lies, when there are four regions. The probability mass function graphically illustrates the confidence with which the region in which the state lies can be identified. For example, when  $P[\mathbf{x} \in r_i] = 1$  and  $P[\mathbf{x} \in r_j] = 0$  for all  $j \neq i$ , then the state is known to be in region  $r_i$ . In contrast, when  $P[\mathbf{x} \in r_i] = \frac{1}{n}$ , where  $n$  is the number of regions in the state space, each of the regions is equally likely to contain the state vector.

The effect of receiving new information  $I_2$  is to conditionally modify the a priori state model  $f_{\mathbf{x}|I}$ , as shown in Figure 3.6. The new information reduces both the uncertainty and the error in the state model. The probability mass function for the region in which the state lies captures the uncertainty in the state model *measured in the context of the proceduralized decision problem*. Section 3.4 will present an approach to modeling the context within which information is valued. This description of the context and the probability mass function for the region in which the state lies are used to define a metric for the cost of the uncertainty in the state



**Figure 3.5. Probability mass function (PMF) for the region in which the state lies.** Three thresholds ( $\theta_1$ ,  $\theta_2$ , and  $\theta_3$ ) define four regions in the state space. The PMF model for the state  $x(t)$  consists of the probabilities the state is in each region, calculated from the probability density function model  $f_{x(t)}$ .

model. This metric will be the basis of the information value definition, presented in Section 3.5. The following section discusses modeling the time-dependence of the state model.



**Figure 3.6. Effect of new information on the model of a state variable.** The effect of receiving new information  $I_2$  is to conditionally modify the a priori model of the state, both adjusting the expected value and reducing the uncertainty.

### 3.3 Modeling Time-Dependence

A measurement describes the condition of a state variable at the time the measurement was taken. As the measurement ages, it less accurately reveals the current or future condition of the state. Consequently, the uncertainty in the state model increases monotonically (i.e., non-decreasing) as the information on which the model is based ages, until the next measurement is received. Figure 3.7 shows two series of state models, predicted at various times after a perfect measurement was taken. To forecast how the state may evolve in the future requires a model of the state dynamics. The two plots in Figure 3.7 are generated by a Kalman filter, which is a model-based estimator, using different models for the state dynamics.

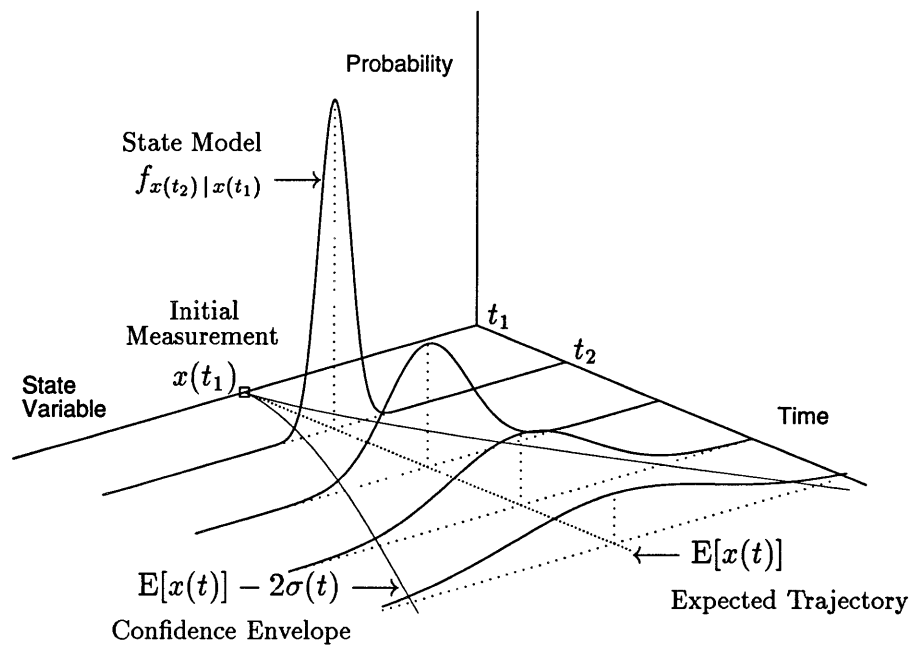


The term *confidence envelope* is used here to define the region in which the state is most likely to fall. In Figure 3.7, there is a 95.5% chance that the state variable is within  $2\sigma$  of the expected value, where  $\sigma$  is the standard deviation and the PDF is normally distributed. Shortly after a measurement, the decision maker is confident that the state lies within a narrow range. However, when the state is predicted further into the future, the decision maker is equally confident only that the state lies within a much broader range. Therefore, to maintain the situation (i.e., state) awareness necessary to make effective procedural decisions, the decision maker must receive repeated observations of the state. Figure 3.8 illustrates the effect of new information (e.g., receiving a measurement taken at a more recent time) on the models of the state at subsequent times. The new information adjusts the expected value of the state and reduces the uncertainty.

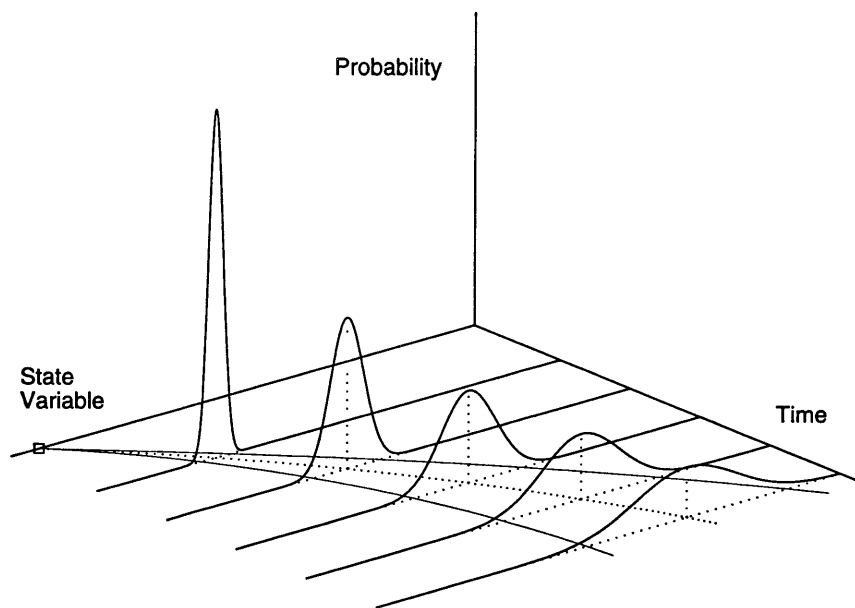
The persistence of information, which might be characterized by a time constant describing the rate at which the variance of the PDF grows, depends on a variety of factors. If the dynamics (or kinematics) of the state variable were known exactly, the future state trajectory could be predicted without any expected error. However, since neither the dynamics nor the external forces acting on the state are known deterministically, the trajectory which the state will follow must be modeled stochastically (i.e., a random process is used to model the state dynamics). The trajectory which the state actually follows will be one sample realization of the random process. The state at a particular time is a random number, whose statistics are determined by the ensemble of the random process at that point in time (Drake, 1967).

Given a PDF for the state at an earlier time, as well as any new information,

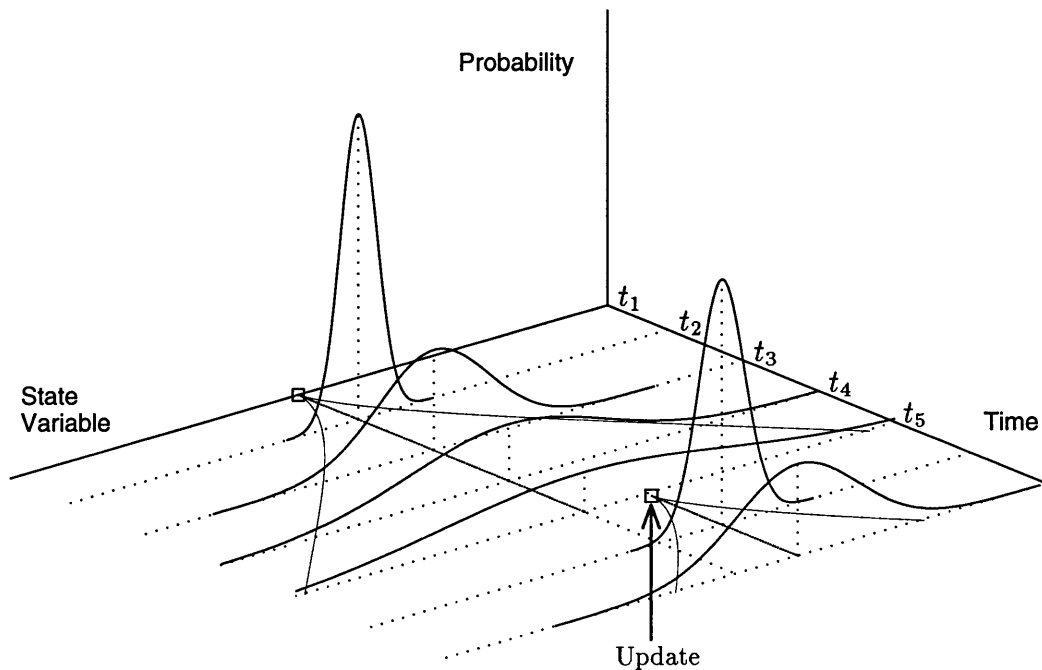
a.



b.



**Figure 3.7. Illustrations of the time-dependence of the state model.** The rate at which uncertainty increases and the evolution of the expected value depend on the model of the state dynamics.



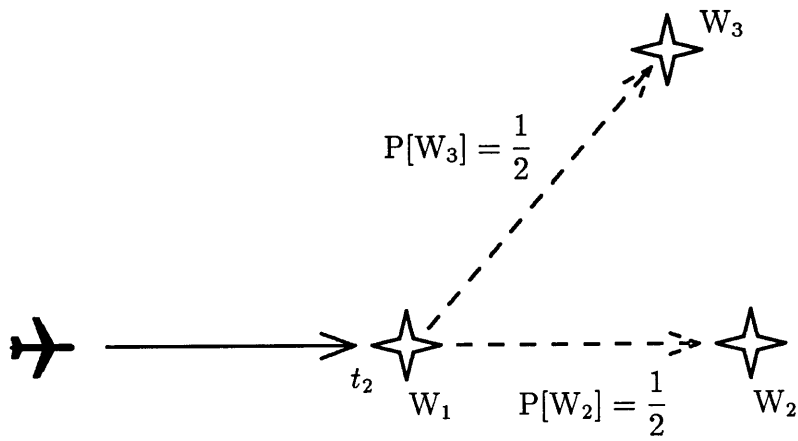
**Figure 3.8. Effect of new information on state models predicted at future times.** The effect of new information (e.g., a measurement) at time  $t_5$  is to adjust the expected value and reduce the expected error.

an estimator based on the model of the state dynamics (e.g., a Kalman filter) is used to construct the probabilistic models (PDF's) of the state at later points in time. Note that if the model of the state dynamics is incorrect, the probability that the actual trajectory lies within the confidence envelope may be small. Therefore, if little is known about the state dynamics, the random process used to model the state dynamics must not be too optimistic with respect to how well the trajectory can be predicted, and, consequently, the persistence of information is very short. This thesis assumes that an analytic model of the random process is available. However, the methodology for valuing information would not change if a Monte Carlo approach was required to construct the PDF's for the state. Refer to Yang and

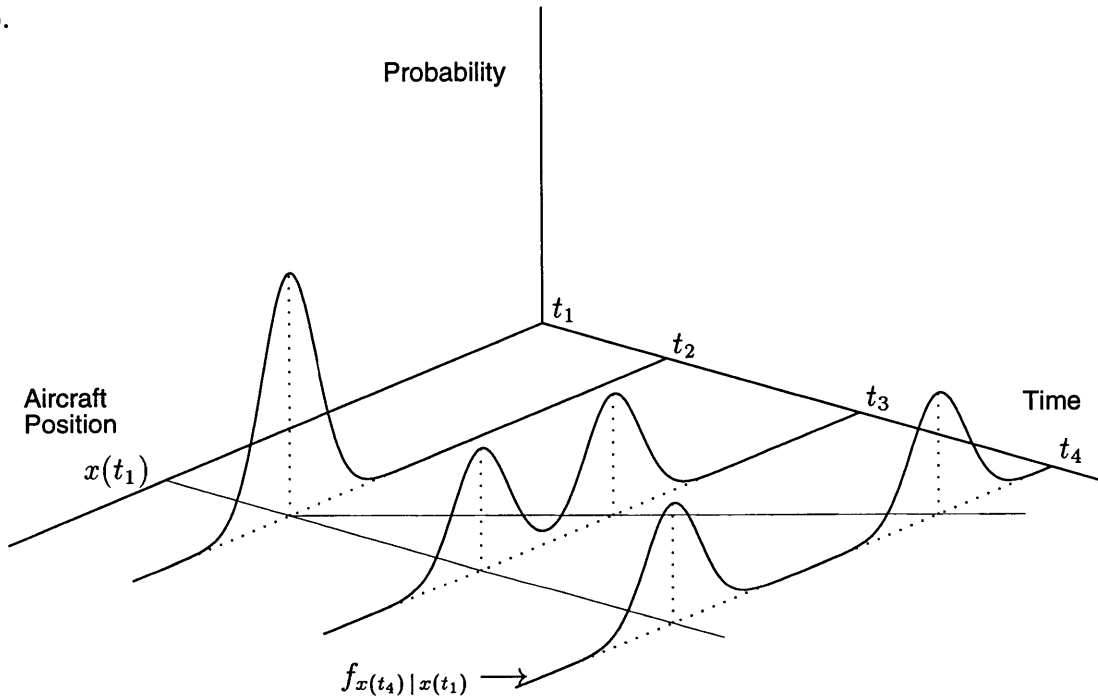
Kuchar (1997) for an example.

In Figures 3.7 and 3.8, the uncertainties in predicting the future state trajectories result from parametric uncertainty in the model of the dynamics or uncertainty in the external inputs. Figures 3.9 illustrates a different source of uncertainty, in which the form of the model of the state dynamics is unknown. Given the available information, an aircraft is considered equally likely to follow either of two airways that depart a waypoint  $W_1$ . Two models for the future state trajectory are feasible; within some tolerance defined by the aircraft's navigation and guidance performance, the aircraft will fly directly to either waypoint  $W_2$  or waypoint  $W_3$ . Part *b* shows models for the aircraft's position at three future times, given this uncertainty in which of the two feasible models for its dynamics is correct.

a.



b.



**Figure 3.9. Uncertainty in the model of the aircraft's future trajectory.** Part a: at time  $t_2$ , the aircraft will reach a waypoint from which two airways diverge. Without information about the aircraft's intent, the aircraft is considered to be equally likely to follow either airway. Part b: models of the aircraft's position at three future times based on information measured at time  $t_1$ .

### 3.4 Modeling Contextual Dependence

Figure 3.2 in Section 3.1 separated the solution to a proceduralized decision problem into two cascaded steps – modeling state variables and applying a decision rule based on that model. The value of information is its effect on the ability to model the state variables, measured in the context of the decision problem. The previous two sections have introduced the time-dependent model of the state variables. This section models the context within which information is valued. Section 3.5 will define the information value metric.

Information cannot be valued independently of the context in which it is applied. For example, information about the temperature in Boston provides little help to the MIT student trying to decide whether or not to carry an umbrella (unless, possibly, it is too cold to rain). However, temperature information is potentially valuable if the student is trying to decide whether or not a warm jacket is required.

The context in which information is valued is not limited to the decision problem to which the information is applied, but may include other aspects of the environment in which the decision is made and the outcome measured. For example, consider a VFR pilot who desires to land at a particular airport but does not know whether fog will cover the airport before he arrives. Assume the original alternate is already closed due to fog. Also assume that if the pilot diverts early, the aircraft has sufficient fuel to reach an alternate airport at which a VFR landing would be permitted. Whether or not the aircraft has a sufficient fuel reserve to reach the alternate airport, after the pilot flies to the primary airport and is unable to land, will affect the value of information about the surface conditions at the primary

airport. Dershowitz (1997) studied how perception of risk and the existence or absence of options affect pilot strategies, supporting the importance of including these aspects of the decision problem, on which the procedure does not explicitly depend, as part of the context within which information is valued.

### 3.4.1 Payoff Matrix

Consider a proceduralized decision problem in which a decision maker must choose between  $M$  actions/decisions  $\{a_1, a_2, \dots, a_M\}$ . Let the state vector  $\mathbf{x} = [x_1, x_2, \dots, x_S]^T$  contain the state variables that are relevant to the procedure. According to the procedure, threshold surfaces divide the  $S$ -dimensional space of relevant state variables into  $n$  regions  $\{r_1, r_2, \dots, r_n\}$ . The *outcome* of decision  $a_j$ , denoted  $\{a_j, r_i\}$ , is the consequence of choosing action  $a_j$  when the state is in region  $r_i$ . The *payoff matrix*  $\kappa$  assigns a cost to every possible outcome, where the cost of outcome  $\{a_j, r_i\}$  is denoted  $\kappa_{ij}$ . For example,  $\kappa_{21}$  is the cost of choosing decision  $a_1$  when the state is in region  $r_2$ . This thesis considers the measure of the decision outcome to be cost (i.e., a larger positive quantity is less favorable) rather than profit. Note that the payoff matrix assumes that the costs  $\kappa_{ij}$  do not depend on where the state lies within the regions. This characteristic results from the definition of a proceduralized decision problem. If the cost of decision outcome  $\{a_j, r_i\}$  is a random number,  $\kappa_{ij}$  will be the expected cost of the outcome.

Tables 3.2 and 3.3 show generic payoff matrices for the cases  $M=2, n=2$  and  $M=3, n=3$ . In general, the elements of the payoff matrix will be time-varying. The implication of the context being time-dependent will be explored in later sections. Table 3.4 summarizes the terminology used in defining the payoff matrix.

**Table 3.2. Payoff matrix,  $M = 2$ ,  $n = 2$ .** The payoff matrix  $\kappa$  assigns a cost to each combination of the action  $a$  and the region  $r$  in which the state  $\mathbf{x}$  lies.

	$a = a_1$	$a = a_2$
$\mathbf{x} \in r_1$	$\kappa_{11}$	$\kappa_{12}$
$\mathbf{x} \in r_2$	$\kappa_{21}$	$\kappa_{22}$

**Table 3.3. Payoff matrix,  $M = 3$ ,  $n = 3$ .**

	$a = a_1$	$a = a_2$	$a = a_3$
$\mathbf{x} \in r_1$	$\kappa_{11}$	$\kappa_{12}$	$\kappa_{13}$
$\mathbf{x} \in r_2$	$\kappa_{21}$	$\kappa_{22}$	$\kappa_{23}$
$\mathbf{x} \in r_3$	$\kappa_{31}$	$\kappa_{32}$	$\kappa_{33}$

Assume that the procedure is optimal with respect to minimizing cost. The *correct* decision for a particular region (i.e., the decision called for by the procedure) is the decision that yields the minimum cost that is achievable when the state lies in that region.  $a_j$  is the correct action, when the state is in region  $r_i$ , if  $\kappa_{ij} = \min_k \kappa_{ik}$ . A decision  $a_j$  is *incorrect* in region  $r_i$  if there exists another possible decision  $a_k$  for which the cost of the decision outcome  $\{a_k, r_i\}$  is less than the cost of the decision outcome  $\{a_j, r_i\}$  (i.e., if  $\kappa_{ik} < \kappa_{ij}$ ). An alternate definition of a *region* is the set of points in the state space for which the same action is correct. Regions are mutually exclusive and collectively exhaustive, as long as there is a single best decision for each point in the state space.



**Table 3.4. Payoff matrix terminology.**

Symbol	Definition
$a_j$	A possible action/decision
$M$	Number of possible actions
$r_i$	A region in the state space
$n$	Number of regions in the state space
$x_k$	A state variable
$\mathbf{x}$	Vector of the state variables relevant to the proceduralized decision problem
$S$	Length of state vector $\mathbf{x}$
$\{a_j, r_i\}$	Decision outcome – the combination of the action that is chosen and the region in which the state lies
$\kappa$	Payoff matrix
$\kappa_{ij}$	Cost of decision outcome $\{a_j, r_i\}$
$\kappa_i^*$	The minimum cost that is achievable when the state is in region $r_i$
$\kappa_{ij} - \kappa_i^*$	Penalty for decision outcome $\{a_j, r_i\}$

Define  $\kappa_i^*$  to be the minimum cost that is achievable when the state is in region  $r_i$  (i.e.,  $\min_j \kappa_{ij}$ ). The *penalty* for a decision outcome  $\{a_j, r_i\}$  is the amount by which the cost for that decision outcome,  $\kappa_{ij}$ , exceeds the cost for the outcome of the correct decision,  $\kappa_i^*$ , (i.e.,  $\kappa_{ij} - \kappa_i^*$ ). Since incorrect decisions are caused by uncertainty in the state model, the penalty is the cost of the uncertainty. The *penalty matrix* gives the penalties for each possible decision outcome (i.e., every combination of action and region).

Table 3.5 shows the penalty matrix for the case  $M = 3$  and  $n = 3$ , and assuming  $a_i$  is the correct action (i.e., the action that minimizes the cost of the

decision outcome) when the state is in region  $r_i$ . For example, when  $\mathbf{x} \in r_1$ , the penalty for choosing action  $a_2$  (an incorrect decision) rather than  $a_1$  (the correct decision) is  $\kappa_{12} - \kappa_{11}$ .

**Table 3.5. Penalty matrix.**

	$a = a_1$	$a = a_2$	$a = a_3$
$\mathbf{x} \in r_1$	0	$\kappa_{12} - \kappa_{11}$	$\kappa_{13} - \kappa_{11}$
$\mathbf{x} \in r_2$	$\kappa_{21} - \kappa_{22}$	0	$\kappa_{23} - \kappa_{22}$
$\mathbf{x} \in r_3$	$\kappa_{31} - \kappa_{33}$	$\kappa_{32} - \kappa_{33}$	0

In general, the penalties are not equal (e.g., when the state is in region  $r_i$ ,  $(\kappa_{ij} - \kappa_i^*) \neq (\kappa_{ik} - \kappa_i^*)$  for  $j \neq k$ ), implying one of the possible incorrect decisions is more costly than the other. Also, the minimum costs are generally not equal (e.g.,  $\kappa_1^* \neq \kappa_2^*$ ), implying it is inherently more costly for the state to be in one region of the state space than in the other.

The payoff matrix is not required to be square. If the number of available actions is larger than the number of regions in the state space (i.e.,  $M > n$ ), then the payoff matrix will have more columns than rows. However, there will be  $M - n$  actions which are not correct for any of the regions. These actions can typically be removed from the payoff matrix, since a decision maker should never choose them. If the number of regions is larger than the number of available actions (i.e.,  $M < n$ ), then at least one action must be correct for multiple regions. However, if the costs resulting from these outcomes are not equal, these regions should not be merged,

and the payoff matrix will remain non-square.

Even if the payoff matrix is square, one of the  $M$  possible actions may be the correct action for more than a single region. In this case, the column of the penalty matrix corresponding to that action would have zeros in multiple rows. If an action is incorrect for every region, the corresponding column of the penalty matrix would have non-zero elements in every row. It is also possible for multiple actions to be correct for the same region, if the costs of the decision outcomes are equal. The corresponding row of the penalty matrix would have multiple zeros.

### 3.4.2 Expected Cost and Expected Uncertainty Cost

The previous section defined the cost  $\kappa_{ij}$  and the penalty  $(\kappa_{ij} - \kappa_i^*)$  for a decision outcome  $\{a_j, r_i\}$  (i.e., given the decision is  $a_j$  and the state is in region  $r_i$ ). This section will use these quantities and a model of the state (i.e., the probabilities the state will be in each of the regions) to construct the random variables *conditional expected decision cost* and *conditional expected uncertainty cost*, given the decision. The subsequent section will introduce a stochastic model for the decision (i.e., the probabilities that each of the available actions will be chosen).

The *conditional expected decision cost*,  $C | a_j$ , is the expected cost of the decision outcome, given the decision is  $a_j$ . The expectation is over the region in which the state lies.

$$C | (I, a_j) = \sum_{i=1}^n P[\mathbf{x} \in r_i | I] \kappa_{ij} \quad \text{for } j \in \{1, 2, \dots, M\} \quad (3.1)$$

For  $n=2$ :

$$C | (I, a_j) = P[\mathbf{x} \in r_1 | I] \kappa_{1j} + P[\mathbf{x} \in r_2 | I] \kappa_{2j} \quad \text{for } j \in \{1, 2, \dots, M\} \quad (3.2)$$

The probabilities  $P[\mathbf{x} \in r_i | I]$  come from the probability mass function  $p_{r|I}$  for the region in which the state lies, calculated in Section 3.2. The available information  $I$  affects the model of the state and, therefore, the probabilities that the state lies in each region. In later sections, the available information will be varied, as a means of calculating the value of information.

Consider the cost that is expected to result from a decision  $a_1$ , when  $n=2$ .

$$C | (I, a_1) = P[\mathbf{x} \in r_1 | I] \kappa_{11} + P[\mathbf{x} \in r_2 | I] \kappa_{21} \quad (3.3)$$

Let  $a_1$  be the correct decision when the state is in region  $r_1$ , which implies  $\kappa_{11} < \kappa_{1j}$  for all  $j \neq 1$ . Also let  $\kappa_{11} > \kappa_{21}$ , which would occur if the state being in region  $r_1$  is inherently more costly than it being in region  $r_2$ . Assume that, given only the initial information  $I$ , there is uncertainty in the model of the state such that the state is predicted to be in each region with equal likelihood (i.e.,  $P[\mathbf{x} \in r_1 | I] = P[\mathbf{x} \in r_2 | I] = \frac{1}{2}$ ). In this case, the conditional expected decision cost equals  $\frac{1}{2} \kappa_{11} + \frac{1}{2} \kappa_{21}$ . Assume new information reveals that the state lies in region  $r_1$  and, therefore, that the original decision  $a_1$  is correct. After receiving this information, the expected decision cost equals  $\kappa_{11}$ . Although the new information reduces the uncertainty in the state model, it increases the expected decision cost. Therefore, expected decision cost is not a useful measure of the consequence of uncertainty in the state model.

Recall that the purpose of providing information is to increase the likelihood that the decision-making agent will make the correct decision, by improving his ability to model the relevant state variables. Therefore, the consequence of the

uncertainty in the state model, measured in the context of the proceduralized decision problem, is the fundamental quantity on which information value will be defined. The concept of *expected uncertainty cost* is introduced as a measure of the consequence of uncertainty in the state model.

The *conditional expected uncertainty cost*,  $R|a_j$ , is the expected penalty for the decision outcome, given the decision is  $a_j$ . Again, the expectation is over the region in which the state lies. “Expected uncertainty cost” should be read as “the expected cost of the uncertainty,” rather than “the cost of the expected uncertainty,” and is interpreted as the expected amount by which the cost exceeds the minimum achievable cost, because the uncertainty in the model of the state may cause the decision-making agent to choose an incorrect decision.

The conditional expected uncertainty cost for the decision  $a_j$  equals the sum, over the  $n$  regions, of the probability that the state  $\mathbf{x}$  will be in region  $r_i$ , weighted by the penalty  $\kappa_{ij} - \kappa_i^*$  for the decision being  $a_j$  when the state is in that region.

$$R| \left( I, a_j \right) = \sum_{i=1}^n P[\mathbf{x} \in r_i | I] (\kappa_{ij} - \kappa_i^*) \quad \text{for } j \in \{1, 2, \dots, M\} \quad (3.4)$$

Given the decision is  $a_j$ , the probability  $P[\mathbf{x} \in r_i | I]$  is the probability that the decision outcome is  $\{a_j, r_i\}$ ; the penalty is the unnecessary cost which results because the decision is incorrect. If the decision  $a_j$  is correct for region  $r_i$ , the penalty equals zero. Notice that the expression for the conditional expected uncertainty cost (3.4) is equivalent to that for the conditional expected decision cost (3.1) with the cost of the decision outcome replaced by the penalty for the decision outcome.

Let  $a_i$  be the correct action when the state is in region  $r_i$ . For  $M=2$  and  $n=2$ :

$$R \mid (I, a) = \begin{cases} P[\mathbf{x} \in r_2 \mid I] (\kappa_{21} - \kappa_{22}) & \text{if } a = a_1, \\ P[\mathbf{x} \in r_1 \mid I] (\kappa_{12} - \kappa_{11}) & \text{if } a = a_2. \end{cases} \quad (3.5)$$

If the action  $a_1$  is chosen, there is probability  $P[\mathbf{x} \in r_2 \mid I]$  that the decision will be wrong.  $\kappa_{21} - \kappa_{22}$  is the penalty for this incorrect decision. Similarly, there is probability  $P[\mathbf{x} \in r_1 \mid I]$  that  $a_2$  would be the wrong decision, and penalty  $\kappa_{12} - \kappa_{11}$  for making that wrong decision.

When  $M=3$  and  $n=3$ : for each decision, there are two regions in which the state can lie for which that decision would be incorrect.

$$R \mid (I, a) = \begin{cases} P[\mathbf{x} \in r_2 \mid I] (\kappa_{21} - \kappa_{22}) + P[\mathbf{x} \in r_3 \mid I] (\kappa_{31} - \kappa_{33}) & \text{if } a = a_1, \\ P[\mathbf{x} \in r_1 \mid I] (\kappa_{12} - \kappa_{11}) + P[\mathbf{x} \in r_3 \mid I] (\kappa_{32} - \kappa_{33}) & \text{if } a = a_2, \\ P[\mathbf{x} \in r_1 \mid I] (\kappa_{13} - \kappa_{11}) + P[\mathbf{x} \in r_2 \mid I] (\kappa_{23} - \kappa_{22}) & \text{if } a = a_3. \end{cases} \quad (3.6)$$

### 3.4.3 Decision Model

The previous section defined the *conditional expected uncertainty cost*, where the conditioning event is the decision which is chosen. This section will use this

quantity and a stochastic model for the decision (i.e., the probabilities that each of the available actions will be chosen) to construct the *expected uncertainty cost*.

To this point, a model for the decision (i.e., which  $a_j$  is chosen) has not been assumed. In general, the decision will be based on the procedure, the probabilities  $P[\mathbf{x} \in r_i]$  that the state is in each of the regions (i.e., the probability mass function  $p_r$ ), and costs for each of the possible decision outcomes (i.e., the payoff matrix). Note that using the probability mass function  $p_r$  allows the decision to depend on the uncertainty about the state as well as the state estimate, measured in the context of the decision problem. A variety of normative models, such as minimum expected cost, are available for describing how an idealized decision maker should choose an action. However, real decision makers are unlikely to conform to these idealized models. Behavioral models for the decision strategy of a particular decision maker are difficult to identify and would be expected to vary from person to person.

Nonetheless, the decision maker's policy for choosing an action affects the value which information will have. Since it is desirable to not limit the definition of information value developed in this thesis to a particular decision policy, such as minimizing the expected cost, a generic stochastic model of the decision is adopted. The decision is modeled by a probability mass function  $p_{a|I}$ , which may be expressed as a vector of probabilities.

$$p_{a|I} = \left[ P[a=a_1 | I], P[a=a_2 | I], \dots, P[a=a_M | I] \right]^T \quad (3.7)$$

$P[a=a_j | I]$  is the probability that action  $a_j$  is chosen, given information  $I$ . The dependence on the information  $I$  comes about because the information is used

to construct the model of the state which is used to make the decision. How the probability mass function is determined from the state model is problem specific, and will be illustrated in the case studies.

Given a probability mass function for the decision, the total expected uncertainty cost  $R \mid I$  may be written as the sum of the probabilities of each action being chosen times the conditional expected uncertainty costs,  $R \mid (I, a)$ , found in the previous section.

$$R \mid I = \sum_{j=1}^M P[a=a_j \mid I] R \mid (I, a_j) \quad (3.8a)$$

$$= \sum_{j=1}^M P[a=a_j \mid I] \sum_{i=1}^n P[\mathbf{x} \in r_i \mid I] (\kappa_{ij} - \kappa_i^*) \quad (3.8b)$$

Let  $a_i$  is the correct decision when the state is in region  $r_i$  (i.e.,  $\kappa_i^* = \kappa_{ii}$ ). Then, for  $M=2, n=2$ :

$$\begin{aligned} R \mid I &= P[a=a_1 \mid I] \times R \mid (I, a_1) \\ &\quad + P[a=a_2 \mid I] \times R \mid (I, a_2) \end{aligned} \quad (3.9a)$$

$$\begin{aligned} &= P[a=a_1 \mid I] P[\mathbf{x} \in r_2 \mid I] (\kappa_{21} - \kappa_{22}) \\ &\quad + P[a=a_2 \mid I] P[\mathbf{x} \in r_1 \mid I] (\kappa_{12} - \kappa_{11}) \end{aligned} \quad (3.9b)$$

For  $M=3, n=3$ :

$$\begin{aligned} R \mid I &= P[a=a_1 \mid I] [P[\mathbf{x} \in r_2 \mid I] (\kappa_{21} - \kappa_{22}) + P[\mathbf{x} \in r_3 \mid I] (\kappa_{31} - \kappa_{33})] \\ &\quad + P[a=a_2 \mid I] [P[\mathbf{x} \in r_1 \mid I] (\kappa_{12} - \kappa_{11}) + P[\mathbf{x} \in r_3 \mid I] (\kappa_{32} - \kappa_{33})] \\ &\quad + P[a=a_3 \mid I] [P[\mathbf{x} \in r_1 \mid I] (\kappa_{13} - \kappa_{11}) + P[\mathbf{x} \in r_2 \mid I] (\kappa_{23} - \kappa_{22})] \end{aligned} \quad (3.10)$$



The expected uncertainty cost will be the basis of the information value measure introduced in Section 3.5. The following section derives an expression for the expected uncertainty cost when the number of regions is infinite. Section 3.4.5 offers an alternate definition for the expected uncertainty cost that is based on a *context matrix* rather than the payoff matrix.

#### 3.4.4 An Infinite Number of Regions

When the number of regions is finite, no significance is attributed to where the state lies within a region. In the limit as the number of regions increases, each “region” will contain only a single point in the state space. The prior expressions (3.4) and (3.8) for the expected uncertainty cost may be rewritten as follows: the action  $a_j$  becomes the decision maker’s estimate of the state  $\hat{x}$ ; the summation, over the  $n$  regions, of the probability that the state is in each region becomes an integral, over the feasible values of the state, of the probability density that the state takes on those values; and the payoff matrix becomes a continuous cost function  $\kappa(x, \hat{x})$ . A single state variable, rather than a vector of state variables, is used to simplify the notation.

$$R \big| (I, \hat{x}) = \int_x f_{x|I}(x' | I) \kappa(x', \hat{x}) dx \quad (3.11)$$

$$R \big| I = \int_{\hat{x}} f_{\hat{x}|I}(\hat{x}' | I) \int_x f_{x|I}(x' | I) \kappa(x', \hat{x}') dx d\hat{x}' \quad (3.12)$$

$f_{\hat{x}|I}$  is the probabilistic model of the decision (i.e., the decision maker’s estimate) and  $f_{x|I}$  is the model of the state. Assume the decision maker chooses the estimate

$\hat{x}$  to be the expected value of the probability density function representing his model of the state,  $E[x]$ , with probability one.

$$f_{\hat{x}|I}(\hat{x}'|I) = \begin{cases} \delta & \text{if } \hat{x}' = E[x], \\ 0 & \text{otherwise.} \end{cases} \quad (3.13)$$

$\delta$  is the unit-impulse function. Then, for the special case  $\kappa(x, \hat{x}) = (x - \hat{x})^2$ ,  $R|I$  may be recognized as the variance of the model of the state.

$$\begin{aligned} R|I &= \int_x f_{x|I}(x'|I) [x' - E[x]]^2 dx \\ &= \sigma_x^2|I \end{aligned} \quad (3.14)$$

### 3.4.5 Context Matrix

Thus far, the thesis has implied (although not required) that the decision maker's model of the state and the true model of the state are equivalent. This section explicitly considers these as two separate models.  $f_{\mathbf{x}_D}$  describes the decision maker's knowledge about the state, which, in the context of the decision problem, may be summarized by the probabilities  $P[\mathbf{x}_D \in r_i]$ .  $f_{\mathbf{x}}$  is the true physical (random) process from which the realized state is a sample realization. Sections 3.2 and 3.3 may be used to construct each of these models, with the model of the state dynamics being different. The subscript D is introduced to identify quantities that are derived from the decision maker's state model  $f_{\mathbf{x}_D}$ . In the calculation of information value, the probability mass function for the decision maker's decision,  $p_a$ ,

**Table 3.6. Context matrix.** The context matrix  $\Gamma$  assigns a cost  $\Gamma_{ij}$  to each combination of the state being in region  $r_i$  when the decision maker estimates that the state is in region  $r_j$ .  $\mathbf{x}_D$  is the decision maker's estimate of the state;  $\mathbf{x}$  is the true state.

	$\mathbf{x}_D \in r_1$	$\mathbf{x}_D \in r_2$
$\mathbf{x} \in r_1$	$\Gamma_{11}$	$\Gamma_{12}$
$\mathbf{x} \in r_2$	$\Gamma_{21}$	$\Gamma_{22}$

depends on the decision maker's state model. The true model of the state is used to calculate the probability mass function for the region in which the state lies,  $p_r$  (i.e., the probabilities  $P[\mathbf{x} \in r_i]$ ).

The previous sections used a payoff matrix, a model of the state, and a model of the decision to define the expected uncertainty cost. The remainder of this section introduces an alternative formulation for the expected uncertainty cost, that replaces the payoff matrix by a *context matrix* and the decision model by the decision maker's state model. The decision rule, which maps the decision maker's state model into the decision, is incorporated within the context matrix. In cases where neither the payoff matrix nor the decision rule are known, assuming an approximate context matrix may be simpler than suggesting separate models for the decision rule and the payoff matrix.

Whereas the payoff matrix describes, for each outcome  $\{a_j, r_i\}$ , the cost of performing action  $a_j$  when the state is in region  $r_i$ , the *context matrix*  $\Gamma$  assigns a cost, in the same units as the payoff matrix, to each combination  $\{i, j\}$  of the state being in region  $r_i$  when the decision maker believes that the state is in region  $r_j$ .

For example, Table 3.6 shows a generic context matrix for the case  $n = 2$ .  $\Gamma_{12}$  is the cost of the decision outcome if  $\mathbf{x} \in r_1$  (i.e., the state is in region  $r_1$ ) and  $P[\mathbf{x}_D \in r_2] = 1$  (i.e., the decision maker believes that the state is in region  $r_2$ ). When  $\mathbf{x} \in r_i$  (i.e., the state is in region  $r_i$ ), the uncertainties in the decision maker's model that are significant to the decision problem are the probabilities  $P[\mathbf{x}_D \in r_j]$ , where  $j \neq i$ . Similar to the previous expression  $(\kappa_{ij} - \kappa_i^*)$  for the penalty for performing action  $a_j$  when the state is in region  $r_i$ , the additional costs resulting from these uncertainties are  $P[\mathbf{x}_D \in r_j](\Gamma_{ij} - \Gamma_{ii})$ , where, by definition,  $\Gamma_{ii}$  is the minimum cost that can be achieved when the state is in region  $r_i$ . Note,  $\Gamma_{ii}$  always equals  $\kappa_i^*$ .

The expected uncertainty cost  $R$  may now be written in terms of the context matrix, as the sum of the probabilities the state is in each region, times the corresponding cost of the uncertainty in the decision maker's state model.

$$R = \sum_{i=1}^n \sum_{j=1}^n P[\mathbf{x} \in r_i] P[\mathbf{x}_D \in r_j] (\Gamma_{ij} - \Gamma_{ii}) \quad (3.15)$$

For  $n=2$ :

$$\begin{aligned} R = & P[\mathbf{x} \in r_1] P[\mathbf{x}_D \in r_2] (\Gamma_{12} - \Gamma_{11}) \\ & + P[\mathbf{x} \in r_2] P[\mathbf{x}_D \in r_1] (\Gamma_{21} - \Gamma_{22}) \end{aligned} \quad (3.16)$$

The state model  $f_{\mathbf{x}_D}$  is the model used to determine the decision. The model  $f_{\mathbf{x}}$  is the model used to calculate the uncertainty with regard to in which region the state lies (i.e., the probability that the decision is incorrect). Often the true

model of the state is not available. Therefore,  $f_{\mathbf{x}}$  should be thought of as the state model of the agent who is calculating the value of information. In general, this agent may be different from the decision maker. When the decision maker and the information-valuing agent are different, the context matrix representation of the expected uncertainty cost provides greater insight into the significance to the value of information of the two models for the state being different. The dependence of information value on the decision policy is more clearly seen in the previous (i.e., payoff matrix) formulation.

The mathematical relationship between the context matrix and the payoff matrix may be found by equating the expressions for the expected uncertainty cost  $R$ , written in terms of the two matrices, and also equating the expressions for the expected decision cost  $C$ , written in terms of the two matrices. The two forms of the definition for the expected uncertainty cost are:

$$R = \sum_{i=1}^n P[\mathbf{x} \in r_i] \sum_{j=1}^n P[\mathbf{x}_D \in r_j] (\Gamma_{ij} - \Gamma_{ii}) \quad (3.17a)$$

$$= \sum_{i=1}^n P[\mathbf{x} \in r_i] \sum_{j=1}^M P[a=a_j] (\kappa_{ij} - \kappa_i^*) \quad (3.17b)$$

For these to be equal, the terms in  $P[\mathbf{x} \in r_i]$  must be equal.

$$\sum_{j=1}^n P[\mathbf{x}_D \in r_j] (\Gamma_{ij} - \Gamma_{ii}) = \sum_{j=1}^M P[a=a_j] (\kappa_{ij} - \kappa_i^*)$$

for all  $i \in \{1, 2, \dots, n\}$  (3.18)

For the case  $n=2$  and  $M=2$ :

$$P[a = a_1] (\kappa_{21} - \kappa_{22}) = P[\mathbf{x}_D \in r_1] (\Gamma_{21} - \Gamma_{22}) \quad (3.19a)$$

$$P[a = a_2] (\kappa_{12} - \kappa_{11}) = P[\mathbf{x}_D \in r_2] (\Gamma_{12} - \Gamma_{11}) \quad (3.19b)$$

The expected decision cost  $C$  may also be written in terms of either the payoff matrix or the context matrix.

$$C = \sum_{i=1}^n P[\mathbf{x} \in r_i] \sum_{j=1}^n P[\mathbf{x}_D \in r_j] \Gamma_{ij} \quad (3.20a)$$

$$= \sum_{i=1}^n P[\mathbf{x} \in r_i] \sum_{j=1}^M P[a=a_j] \kappa_{ij} \quad (3.20b)$$

For the case  $n=2$  and  $M=2$ :

$$\begin{aligned} C &= P[\mathbf{x} \in r_1] \left( P[a=a_1] \kappa_{11} + P[a=a_2] \kappa_{12} \right) \\ &\quad + P[\mathbf{x} \in r_2] \left( P[a=a_1] \kappa_{21} + P[a=a_2] \kappa_{22} \right) \end{aligned} \quad (3.21a)$$

$$\begin{aligned} &= P[\mathbf{x} \in r_1] \left( P[\mathbf{x}_D \in r_1] \Gamma_{11} + P[\mathbf{x}_D \in r_2] \Gamma_{12} \right) \\ &\quad + P[\mathbf{x} \in r_2] \left( P[\mathbf{x}_D \in r_1] \Gamma_{21} + P[\mathbf{x}_D \in r_2] \Gamma_{22} \right) \end{aligned} \quad (3.21b)$$

In order for the two expressions for  $C$  to be equal for all values of  $P[\mathbf{x} \in r_i]$ , the terms in  $P[\mathbf{x} \in r_i]$  must be individually equal.

$$\sum_{j=1}^n P[\mathbf{x}_D \in r_j] \Gamma_{ij} = \sum_{j=1}^M P[a=a_j] \kappa_{ij} \quad \text{for all } i \in \{1, 2, \dots, n\} \quad (3.22)$$

For the case  $n=2$  and  $M=2$ :

$$P[a=a_1] \kappa_{11} + P[a=a_2] \kappa_{12} = P[\mathbf{x}_D \in r_1] \Gamma_{11} + P[\mathbf{x}_D \in r_2] \Gamma_{12} \quad (3.23a)$$

$$P[a=a_1] \kappa_{21} + P[a=a_2] \kappa_{22} = P[\mathbf{x}_D \in r_1] \Gamma_{21} + P[\mathbf{x}_D \in r_2] \Gamma_{22} \quad (3.23b)$$

Given either the payoff matrix  $\kappa$  or the context matrix  $\Gamma$ , the system of linear equations specified by (3.18) and (3.22) may be solved to find the other matrix. For example, given a 2 by 2 payoff matrix  $\kappa$ , equations (3.19) and (3.23) may be solved simultaneously for the four unknown elements of the context matrix. (3.19b) and (3.23a) may be solved to find  $\Gamma_{12}$  and  $\Gamma_{11}$ , which are decoupled from  $\Gamma_{21}$  and  $\Gamma_{22}$ .  $\Gamma_{21}$  and  $\Gamma_{22}$  are found from (3.19a) and (3.23b). Using the fact that  $P[a = a_1] + P[a = a_2] = 1$  and  $P[\mathbf{x}_D \in r_1] + P[\mathbf{x}_D \in r_2] = 1$ , the following relationships may be determined.

$$\Gamma_{11} = \kappa_{11} \quad (3.24a)$$

$$\Gamma_{12} = \left( \frac{P[a = a_1] - P[\mathbf{x}_D \in r_1]}{P[\mathbf{x}_D \in r_2]} \right) \kappa_{11} + \frac{P[a = a_2]}{P[\mathbf{x}_D \in r_2]} \kappa_{12} \quad (3.24b)$$

$$\Gamma_{22} = \kappa_{22} \quad (3.24c)$$

$$\Gamma_{21} = \frac{P[a = a_1]}{P[\mathbf{x}_D \in r_1]} \kappa_{21} + \left( \frac{P[\mathbf{x}_D \in r_1] - P[a = a_1]}{P[\mathbf{x}_D \in r_1]} \right) \kappa_{22} \quad (3.24d)$$

### 3.4.6 The Perspective from which Information is Valued

If the decision maker himself is measuring his self-perceived value of information, he will use his model of the state to both determine his decision and measure the uncertainty with regard to in which region the state lies.

Alternatively, the value which information has to the decision maker may be measured by an external *information-valuing agent*. For example, if the decision maker is a pilot, the information-valuing agent might be the operator of a ground-based sensor. The agent may also be a piece of automation which has the

responsibility for determining when a new measurement should be communicated to the pilot. The information-valuing agent may have a different model of the state, with which it calculates the probabilities the state is in each of the regions, from the decision maker. However, since a model of the decision is required to value information and the decision depends on the decision maker's model of the state, the information-valuing agent must also have a model of the decision maker's state model.

In the expected uncertainty cost construct based on the payoff matrix, the decision maker's state model is hidden within the model of the decision. When the information-valuing agent's state model is different than the decision maker's state model, the expression for expected uncertainty cost based on the context matrix allows both state models to be seen explicitly, although at the cost of hiding the decision rule.

If the goal for applying this thesis is to design a process to manage information dissemination, it is useful to think of the information management process as the information-valuing agent, and the agent's model of the state  $f_{\mathbf{x}}$  as describing the true random process for the state (or the best model that is available).

## 3.5 Definition of Information Value

### 3.5.1 Conditional Information Value

Given initially available information  $I_1$ , a model of the relevant state variables  $f_{\mathbf{x}|I_1}$  is constructed. In the context of the proceduralized decision, the a



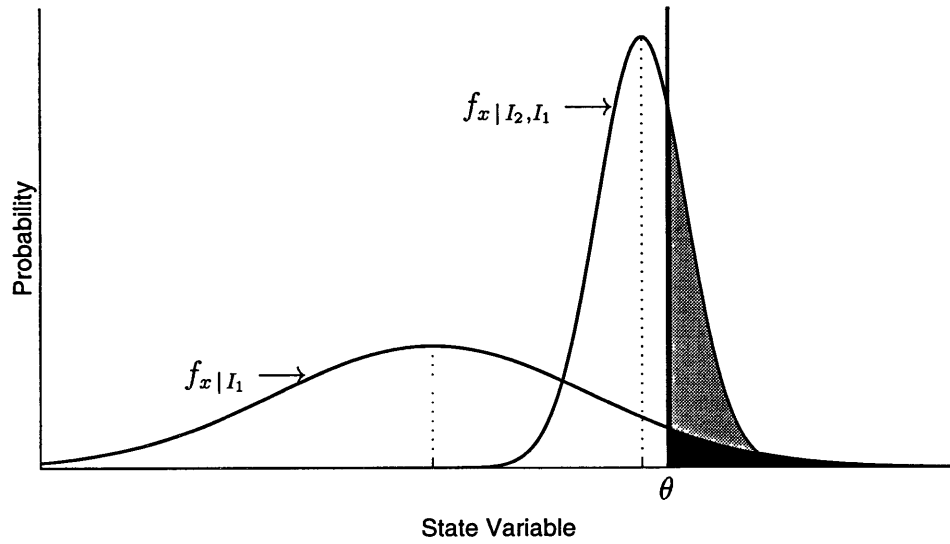
priori expected uncertainty cost (i.e., the expected additional cost that will be incurred because of the uncertainty in the state model) is  $R | I_1$ . Sections 3.2 and 3.4 discussed these steps. Given new information  $I_2$ , the a posteriori expected uncertainty cost is  $R | (I_2, I_1)$ . Note that the original information  $I_1$  may still contribute to the a posteriori model of the state,  $f_{\mathbf{x}} | I_2, I_1$ .

The *conditional information value*,  $V | I_2$ , given that the new information is  $I_2$ , equals the magnitude of the change in the expected uncertainty cost.

$$V | I_2 = \left| R | I_1 - R | (I_2, I_1) \right| \quad (3.25)$$

The conditional information value measures the effect of information  $I_2$  on the model of the state, in the context of the proceduralized decision. Conditional information value is conditioned on what information is received. The following section will define the *expected information value*, when the content of the information is not known (i.e., before receiving the information).

Information should have positive value when it allows the true state to be estimated or predicted more accurately and confidently, and that change in the state model is significant to the decision problem (i.e., increases the likelihood that the correct decision will be chosen), even if receiving the information increases the expected cost of the decision. By more accurately revealing the true state of the world, additional information is generally expected to reduce the expected uncertainty cost. However, new information may change the expected value of the state, in addition to reducing the variance of the state model. If the true state, revealed by the information, is closer to a threshold between two regions than the



**Figure 3.10.** New information  $I_2$  that increases the expected uncertainty cost.

expected value of the a priori state model, then the information may increase the expected uncertainty cost, as shown in Figure 3.10.

A positive value is attributed to information that increases the expected uncertainty cost because it reveals: (1) that the a priori state model was misleadingly confident about how well the region in which the state lies is known, (2) that this larger uncertainty should be taken into account when the decision is chosen, and (3) that additional information should be sought to reduce the expected uncertainty cost. The absolute value function is included in the definition of information value to positively value all changes in expected uncertainty cost.

### 3.5.2 Expected Information Value

Often, the decision whether or not to seek the new information must be made prior to knowing the content of the information. ‘Should a measurement be taken?’

must be answered before what measurement will be received can be known. Similarly, a decision maker must decide whether or not to request that a piece of information be communicated across a datalink before the content of the information is known. The *expected information value* is the expectation of the *conditional information value* over the information that may be received.

Consider a proceduralized decision that depends on the condition of a single state variable  $x$  at a future time  $t_3$ , with respect to a threshold  $\theta$ , as shown in Figure 3.10. At time  $t_1$ , the decision maker receives a perfect measurement of the state at that time,  $x(t_1)$ .<sup>2</sup> The threshold  $\theta$  distinguishes two regions in the state space. Let region  $r_1$  be the set  $x(t_3) > \theta$  and region  $r_2$  be the set  $x(t_3) < \theta$ . At any time  $t_2$ , between times  $t_1$  and  $t_3$ , the decision maker selects an action  $a(t_2) \in \{a_1, a_2\}$ . In this case, the payoff matrix is 2 by 2.

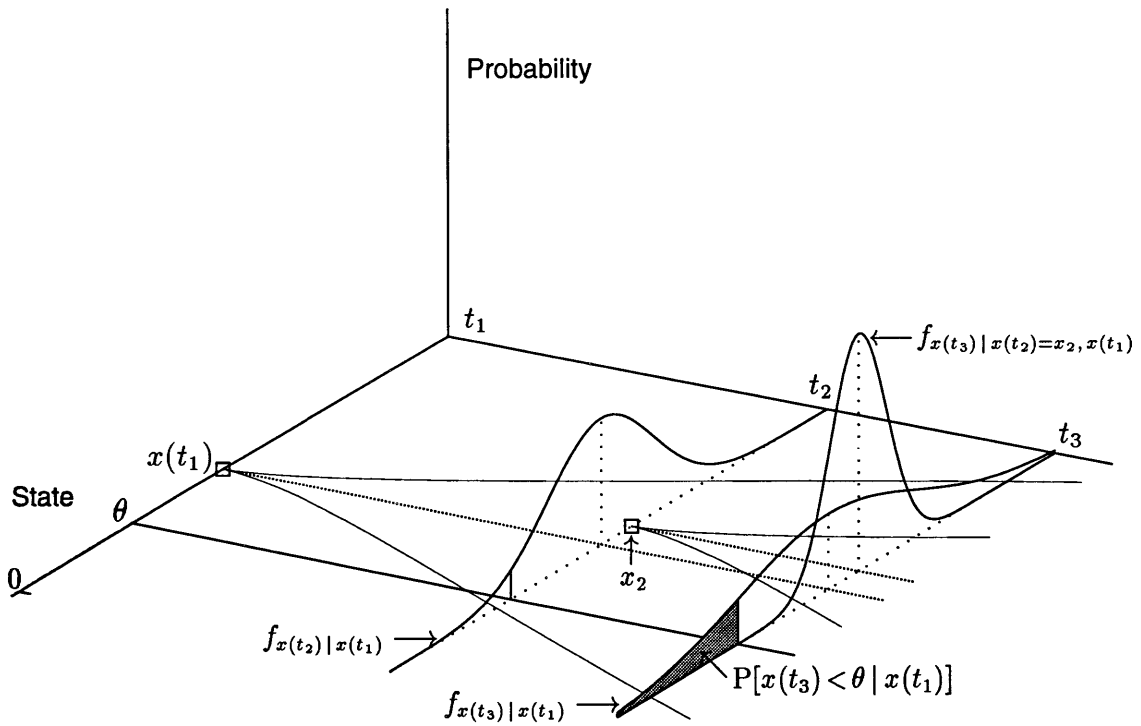
Figure 3.11 shows models of the state based on various information. The probability density function  $f_{x(t_3)|x(t_1)}$  is the model of the state at time  $t_3$ , based only on the initial information – that the state at time  $t_1$  is  $x(t_1)$ . Given the measurement at time  $t_2$  is  $x_2$ , the model of the state at time  $t_3$  becomes  $f_{x(t_3)|x(t_2)=x_2, x(t_1)}$ , drawn in the figure for one possible measurement. Note that receiving the new information may cause the decision maker to switch the decision from that which he would have made with only the original information. However, prior to receiving the new measurement, the decision maker does not know what the measurement will be. The a priori model for the state at  $t_2$  is  $f_{x(t_2)|x(t_1)}$ . This probability density function is the distribution for the measurement at  $t_2$ , assuming there is no mea-

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<sup>2</sup> Appendix B discusses state estimation when the information includes a measurement error, or the state of interest must be constructed from measurements of other state variables.

**Table 3.7. Terminology used in information value definition.**

Symbol	Definition
$x(t_1)$	The state at time $t_1$ .
$x_2$	The measurement that is received at $t_2$ .
$f_{x(t_2) x(t_1)}$	The model of the state variable at time $t_2$ , based on the initial information $x(t_1)$ . The PDF for the random variable $x_2$ .
$f_{x(t_3) x(t_1)}$	The model of the state variable at time $t_3$ , based on the initial information $x(t_1)$ .
$f_{x(t_3) x(t_2)=x_2, x(t_1)}$	The model of the state variable at time $t_3$ , based on the information that the state is $x(t_1)$ at time $t_1$ and $x_2$ at time $t_2$ .
$P[x(t_3) < \theta   x(t_1)]$	The probability that at time $t_3$ the state variable is less than the threshold $\theta$ , calculated using the model $f_{x(t_3) x(t_1)}$ .



**Figure 3.11. Illustration of the information value definition.**

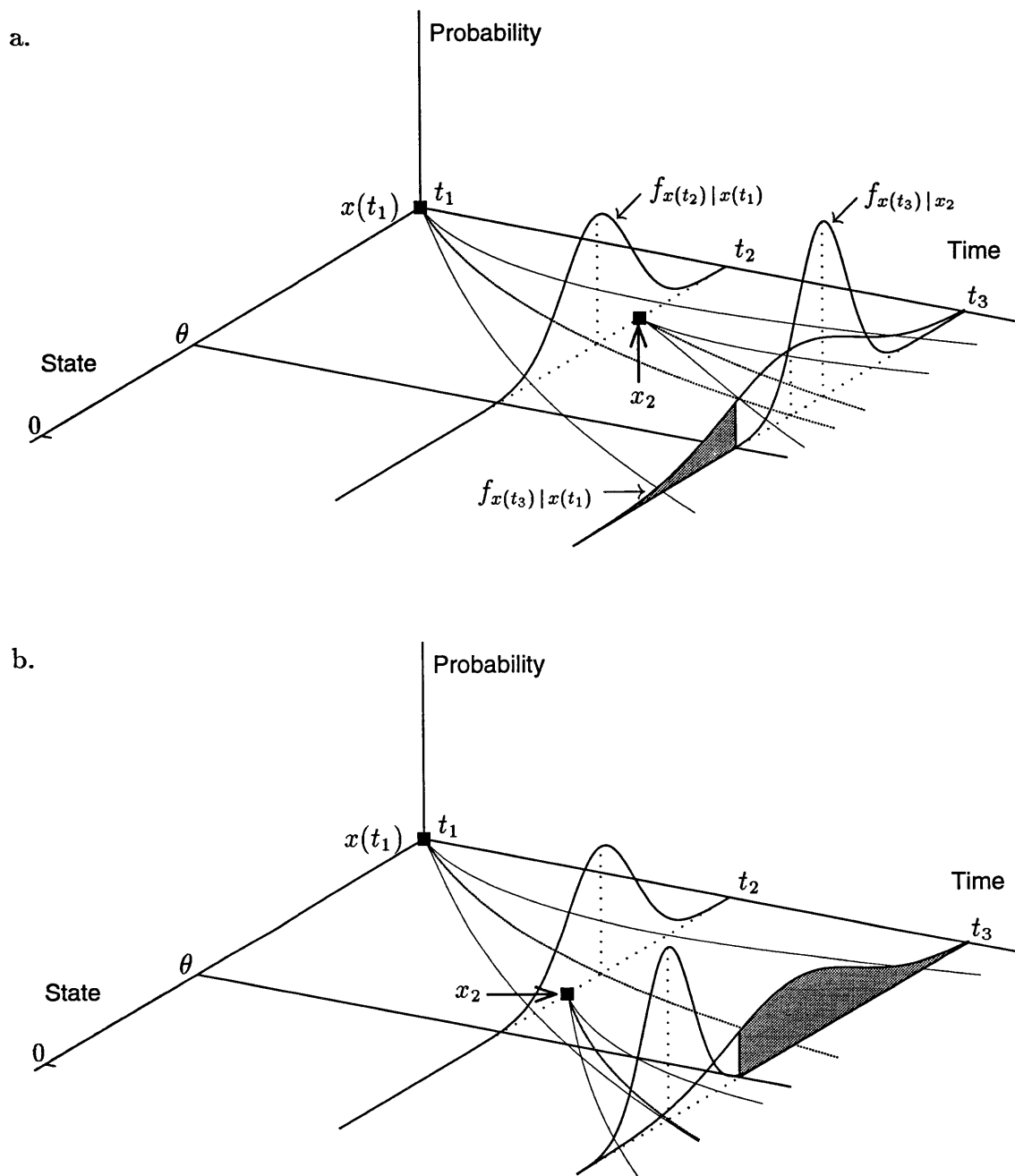
surement error. Appendix B shows the effect of sensor performance (i.e., imperfect measurements) on the distribution for the measurement. Table 3.7 summarizes the terminology used in Figure 3.11.

Figure 3.12 illustrates the effect of the measurement at time  $t_2$  taking on two different values, on the model of the state at time  $t_3$ . Although in both of the cases shown in the figure the measurement reduces the uncertainty about in which region  $x(t_3)$  will lie, the a posteriori models of  $x(t_3)$  are very different and the decisions that would be made after those measurements are unlikely to be the same.

The expected value of the measurement at time  $t_2$ ,  $V$ , is defined as the expectation over the feasible measurements of the values for those measurements.

$$V(x(t_1), t_2) = \int_{x_2} f_{x(t_2)|x(t_1)}(x_2 | x(t_1)) \left| R|x(t_1) - R|(x(t_2)=x_2, x(t_1)) \right| dx_2 \quad (3.26)$$

The dependence of  $V$  on  $t_2$  is shown explicitly, as a reminder that information value depends on the time at which the information is measured. The value of information also depends on the initial state and, implicitly, on the model of the state dynamics. The following section demonstrates the calculation of expected information value and discusses typical results.



**Figure 3.12.** The effect of the measurement taking on different values. Part a:  $P[x(t_3) < \theta | x_2] \approx 0$ . Part b:  $P[x(t_3) < \theta | x_2] \approx 1$ . In both cases, the measurement reduces the uncertainty with respect to in which region  $x(t_3)$  will lie.

## 3.6 Example Results

### 3.6.1 Problem Formulation

Assume that at time  $t_2$  ( $t_1 < t_2 < t_3$ ) a proceduralized decision must be made between two actions;  $a(t_2) \in \{a_1, a_2\}$ . Let  $t_1 = 0$  and  $t_3 = 1$ . The procedure depends on the condition, relative to a threshold  $\theta$ , of a state variable  $x$  at future time  $t_3$ . According to the procedure, action  $a_1$  should be chosen when  $x(t_3) < \theta$  and action  $a_2$  should be chosen otherwise. Let  $\theta = 0$ . Assume the symmetric payoff matrix shown in Table 3.8. The costs for the correct decisions ( $\kappa_{11}$  and  $\kappa_{22}$ ) are equal, implying that neither region of the state space is more costly than the other. Moreover,  $\kappa_{11}$  and  $\kappa_{22}$  increase with increasing  $t_2$ . This time-dependence represents an opportunity cost for delaying the decision. The costs for the incorrect decisions ( $\kappa_{12}$  and  $\kappa_{21}$ ) are equal and constant, such that when  $t_2 = t_3$ , the cost for a correct decision equals the cost for an incorrect decision. Although the cost resulting from the decision increases to a maximum value at  $t_2 = t_3$ , the penalties for the incorrect decisions, given by Equation (3.27), are equal and decrease from a maximum of one at  $t_2 = t_1$  to zero at  $t_2 = t_3$ . Consequently, because delaying the decision erodes the benefit of making the correct decision, information will have zero value when  $t_2 = t_3$ , even if the new information increases the likelihood that a correct decision will be made.

$$\kappa_{12} - \kappa_{11} = 1 - \frac{t_2 - t_1}{t_3 - t_1} \quad (3.27a)$$

$$\kappa_{21} - \kappa_{22} = 1 - \frac{t_2 - t_1}{t_3 - t_1} \quad (3.27b)$$

**Table 3.8. Payoff matrix for the example problem.**

	$a(t_2) = a_1$	$a(t_2) = a_2$
$x(t_3) > \theta$	$\frac{t_2 - t_1}{t_3 - t_1}$	1
$x(t_3) < \theta$	1	$\frac{t_2 - t_1}{t_3 - t_1}$

The state variable  $x(t)$  is modeled as the output of an integrated, first-order Markov process (Brown, 1983), with the parameters  $\sigma=4$  and  $\beta=0.1$ . Appendix A describes this random process. The state at a future time is modeled as a normally distributed random variable, where the estimate and variance are predicted by a Kalman filter. Appendix A also describes the Kalman filter algorithm. Repeating Equations (3.8) and (3.26), the expected value of information  $V$  is given by:

$$V(x(t_1), t_2) = \int_{x_2} f_{x(t_2)|x(t_1)}(x_2 | x(t_1)) \times \left| R | x(t_1) - R | (x(t_2)=x_2, x(t_1)) \right| dx_2 \quad (3.28)$$

where the expected cost of uncertainty  $R$  is:

$$\begin{aligned} R | I &= P[a(t_2)=a_1 | I] P[x(t_3) < \theta | I] (\kappa_{21} - \kappa_{22}) \\ &+ P[a(t_2)=a_2 | I] P[x(t_3) > \theta | I] (\kappa_{12} - \kappa_{11}) \end{aligned} \quad (3.29)$$

and the available information  $I$  may either be limited to the initial measurement  $x(t_1)$  or may also include the second measurement  $x(t_2)$ . Note that receiving the



new information will not only change the decision maker's uncertainty as to whether or not the chosen action is correct, but may also change the action which the decision maker chooses.

A deterministic model of the decision maker's decision policy is generally not available. Therefore, a stochastic description of the decision (i.e., the probabilities of each action being chosen) is used to model the decision. In this example, the probabilities of each action being chosen are assumed to equal the probabilities that  $x(t_3)$  lies in the corresponding regions.

$$P[a(t_2)=a_1 | I] = P[x(t_3) > \theta | I] \quad (3.30a)$$

$$P[a(t_2)=a_2 | I] = P[x(t_3) < \theta | I] \quad (3.30b)$$

Note that this is the decision rule for which the payoff matrix and context matrix are equal. Using this model for the decision policy, (3.28) and (3.29) may be rewritten. Although this example is dimensionless, in general, information value is measured in the units of the payoff matrix.

$$R | I = P[x(t_3) > \theta | I] P[x(t_3) < \theta | I] (\kappa_{21} - \kappa_{22} + \kappa_{12} - \kappa_{11}) \quad (3.31)$$

$$= P[x(t_3) > \theta | I] (1 - P[x(t_3) > \theta | I]) (\kappa_{21} - \kappa_{22} + \kappa_{12} - \kappa_{11})$$

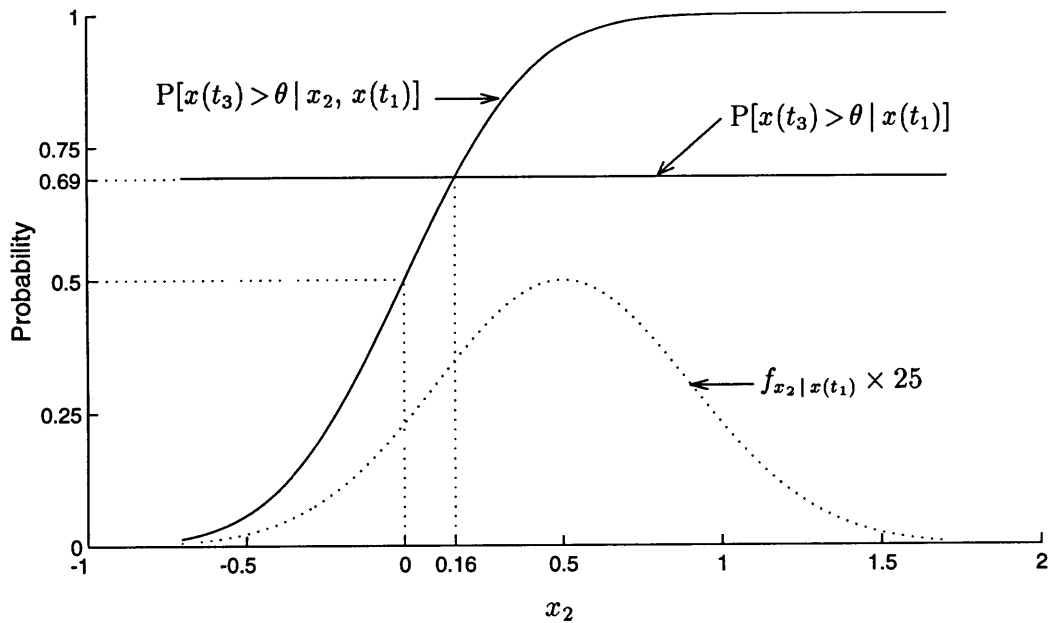
$$\begin{aligned} V(x(t_1), t_2) &= (\kappa_{21} - \kappa_{22} + \kappa_{12} - \kappa_{11}) \int_{x_2} f_{x(t_2)|x(t_1)}(x_2 | x(t_1)) \times \\ &\quad \left| P[x(t_3) > \theta | x(t_1)] (1 - P[x(t_3) > \theta | x(t_1)]) - \right. \\ &\quad \left. P[x(t_3) > \theta | x_2, x(t_1)] (1 - P[x(t_3) > \theta | x_2, x(t_1)]) \right| dx_2 \end{aligned} \quad (3.32)$$

### 3.6.2 Results

A new measurement  $x_2$  taken at time  $t_2$  changes the model of the state at time  $t_3$  from  $f_{x(t_3)|x(t_1)}$  to  $f_{x(t_3)|x_2, x(t_1)}$ . Equation (3.32) shows that, in this example, the value of receiving the new measurement is related to the resulting change in the probability that the state at time  $t_3$  will be greater than the threshold  $\theta$ . The measurement  $x_2$  changes this probability from  $P[x(t_3) > \theta | x(t_1)]$  to  $P[x(t_3) > \theta | x_2, x(t_1)]$ , where  $P[x(t_3) > \theta | I]$  equals the integral of the PDF  $f_{x(t_3)|I}$  over the region  $x(t_3) > \theta$ . These probabilities are plotted in Figure 3.13 as a function of the measurement  $x_2$ , for the case  $x(t_1) = 0.5$  and  $t_2 = 0.5$ . Recall that  $\theta = 0$  and  $t_3 = 1$ . The a priori probability is a constant. The probability density function for  $x_2$  is also shown, scaled to be readable on the same axis.

Prior to receiving the new measurement, the probability that the state will be greater than the threshold at time  $t_3$ ,  $P[x(t_3) > \theta | x(t_1)]$ , is 0.69. If the new measurement  $x_2$  is greater than 0.16, the state is more likely to be greater than the threshold at time  $t_3$  (i.e.,  $P[x(t_3) > \theta | x_2, x(t_1)] > 0.69$ ). If the measurement  $x_2$  is near the threshold ( $\theta = 0$ ), then the probability that the state will be greater than the threshold at time  $t_3$  is closer to 0.5 after receiving the measurement. Therefore, the region in which the state will lie is more uncertain than originally thought. Measurements in this range are valued positively, because this larger uncertainty should be taken into account when making the decision.

Figure 3.14 shows the expected value of new information  $V(x(t_1), t_2)$  as a function of the time  $t_2$  at which the information is measured, for several values of the initial state  $x(t_1)$ . The value of  $t_2$  for which expected information value is largest varies with  $x(t_1)$  and, as would be expected, has higher peak values when



**Figure 3.13.** The effect of a new measurement on the probability the state will be greater than the threshold at time  $t_3$ , as a function of the measurement. For the case  $x(t_1) = 0.5$  and  $t_2 = 0.5$ .  $\theta = 0$ .  $t_3 = 1$ .

the initial value of the state is near the threshold (i.e., when initial uncertainty about the region in which the state will lie is high). For each value of  $x(t_1)$ , the time  $t_2$  that maximizes expected information value is a tradeoff between making the decision at an earlier time, when the cost of a correct decision is smaller due to the time-dependence of the payoff matrix, and making the decision at a later time, when the new measurement will allow a more accurate prediction of whether the state at time  $t_3$  will be greater or less than the threshold.

Figures 3.15 and 3.16 illustrate the effect of the new measurement being taken at early and late times. In Figure 3.15, the new measurement taken at an early time has little effect on the model of  $x(t_3)$ . Figure 3.16 shows how a new measurement taken at a late time may allow the decision maker to predict, with a high probability

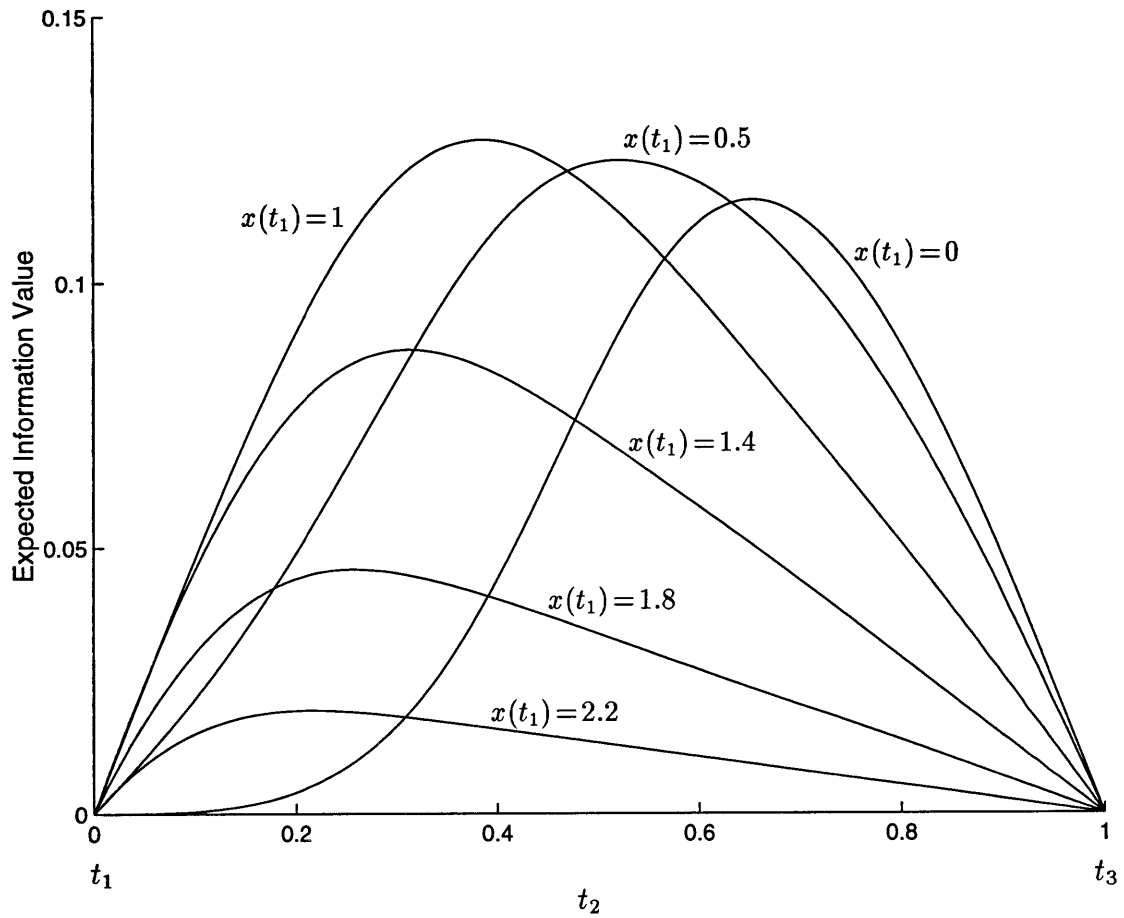
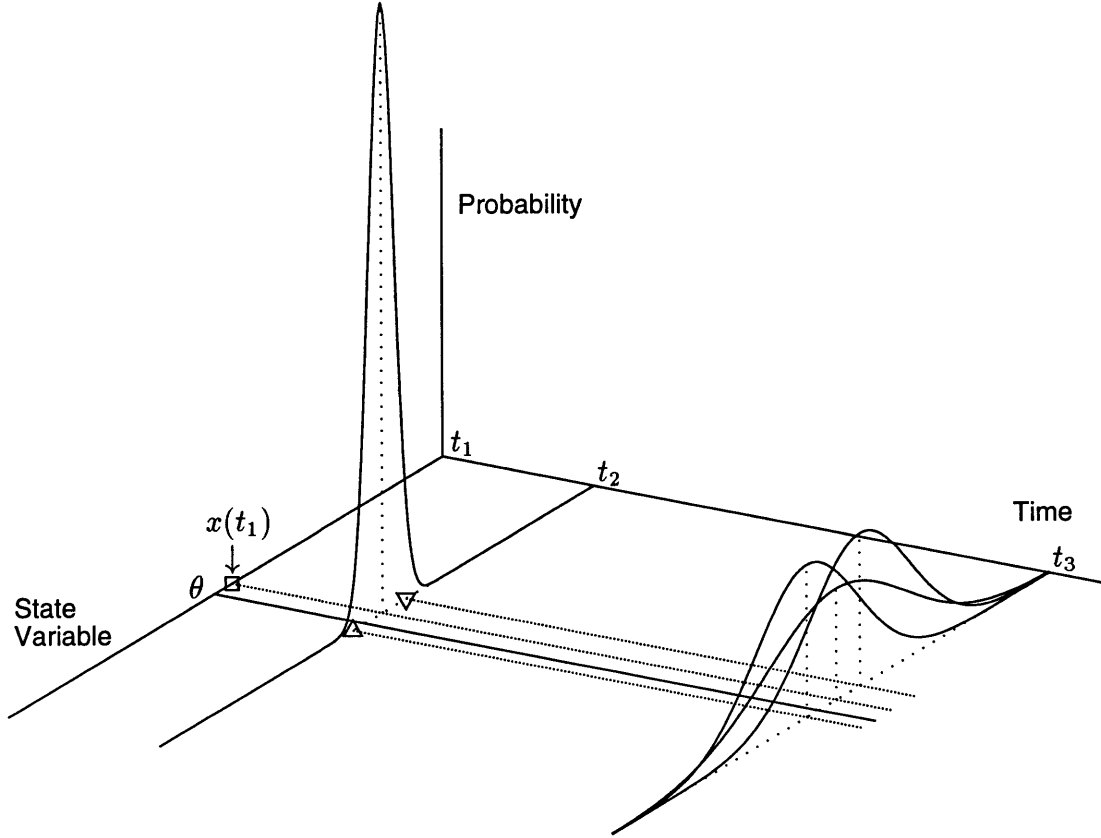


Figure 3.14. Expected information value, plotted as a function of the time at which the information is measured, for several values of the initial measurement.  $\theta=0$ .

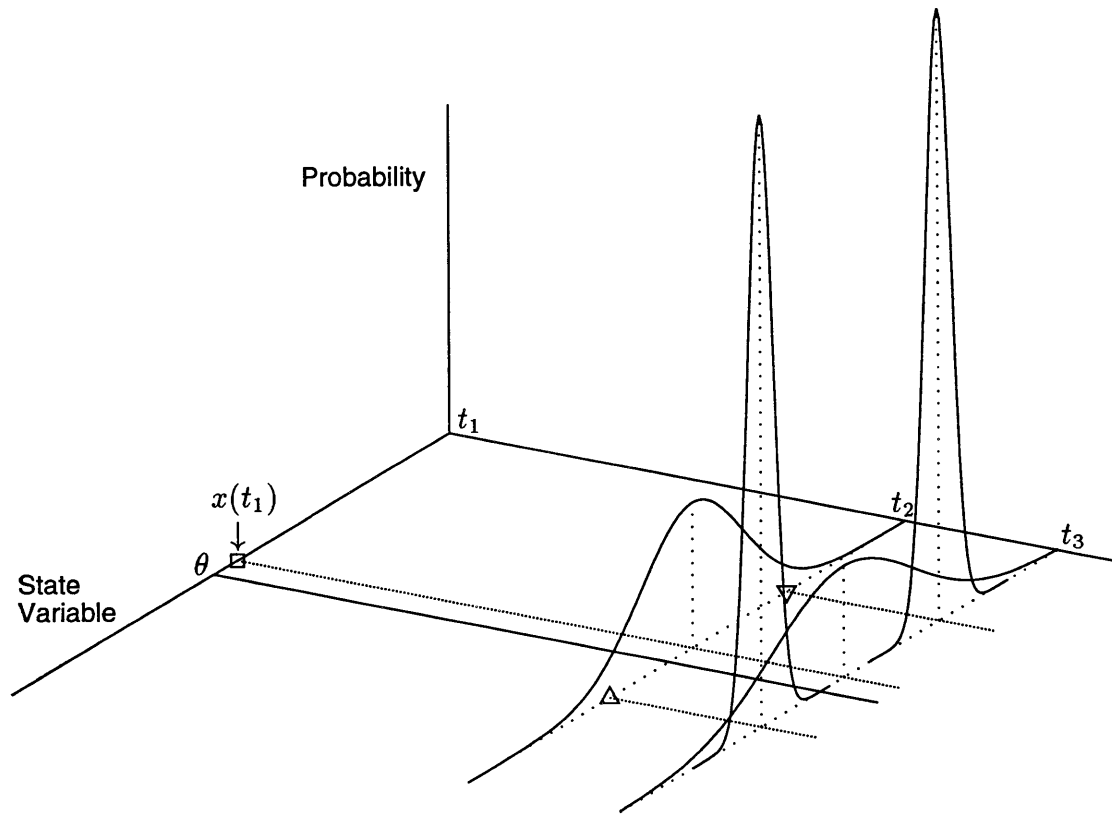
of being correct, whether  $x(t_3)$  will be greater or less than  $\theta$ , even when the state is near the threshold at time  $t_1$ . However, prior to receiving the measurement, the decision maker does not know whether it will be greater or less than the threshold.

In order for a measurement that falls closer to the threshold to provide the same improvement in the confidence with which whether  $x(t_3)$  will be greater or less than the threshold may be predicted, it must have been measured at a later time than a measurement that falls farther from the threshold. Alternatively, for any  $t_2$ ,



**Figure 3.15.** The effect of the new measurement being taken at an early time. The feasible measurements at time  $t_2$  fall within a narrow range. The figure shows two possible measurements (represented by  $\Delta$  and  $\nabla$ ) neither of which significantly reduces the uncertainty in the model for the state at time  $t_3$ .

moving the initial state farther from the threshold improves the ability to predict whether  $x(t_3)$  will be greater or less than the threshold. In this example, the new measurement is likely to fall near the state at time  $t_1$ . Consequently, when  $x(t_1)$  is large (i.e., far from the threshold), the new measurement has maximum value when  $t_2$  is small, because an early decision can be made with a high probability of being correct. As  $x(t_1)$  is reduced (i.e., approaches the threshold), the value of  $t_2$  for which information value is greatest increases.



**Figure 3.16.** The effect of the new measurement being taken at a late time. The feasible measurements at time  $t_2$  (two are represented by  $\Delta$  and  $\nabla$ ) occupy a wide range. Given either measurement, the resulting model for the state at  $t_3$  has a small expected error and a high probability of correctly identifying the region in which the state will lie.

The maximum values of information (with respect to  $t_2$ ) are largest when  $x(t_1)$  is near the threshold (i.e., when the initial uncertainty concerning whether  $x(t_3)$  will be greater or less than the threshold is largest). For information to have value, uncertainty that is significant to the decision problem must initially exist. When  $x(t_1)$  is far from the threshold, relative to the rate at which the state can change, the uncertainty (i.e., variance) in the model of the state at time  $t_3$  has little significance to the decision problem. However, information does not have its

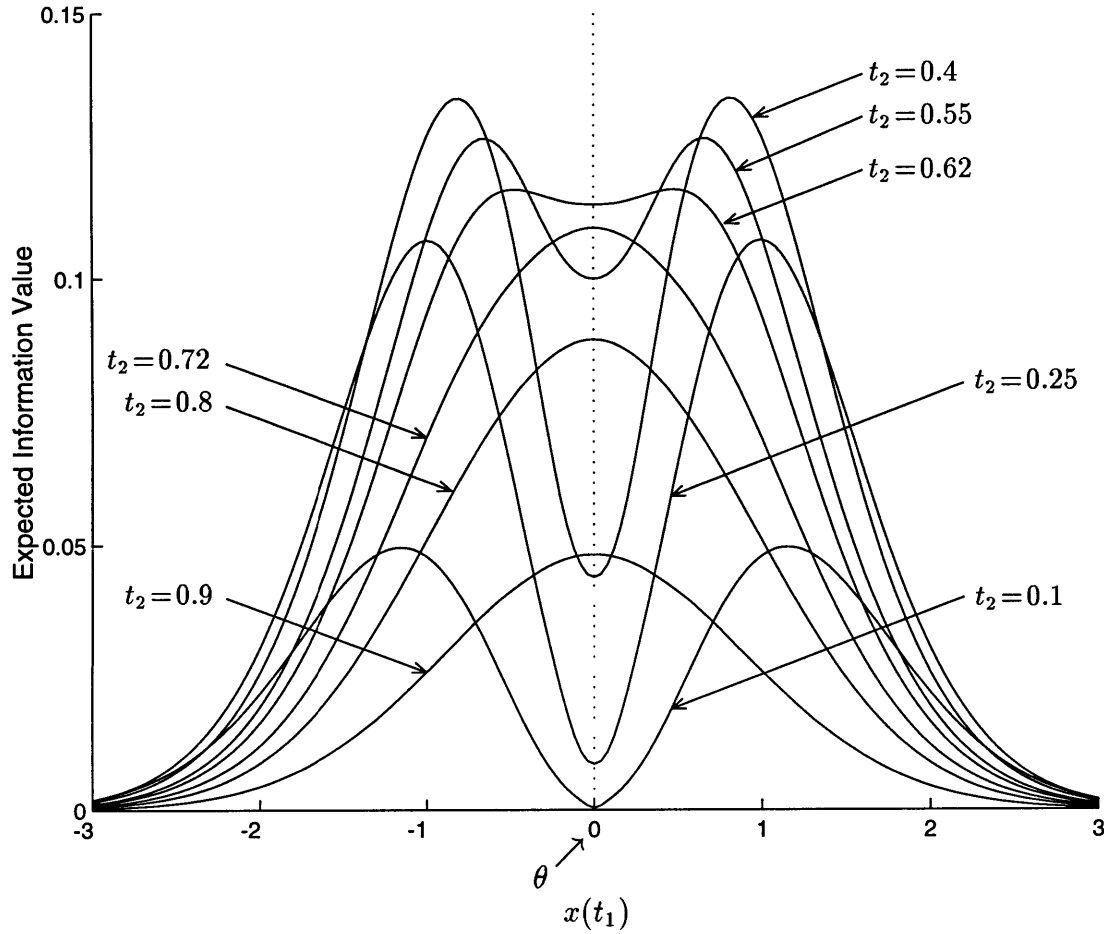


Figure 3.17. Expected information value, plotted as a function of the initial measurement, for several value of the time at which the information is measured.

overall largest value when the initial state is exactly equal to the threshold. In order to accurately predict whether  $x(t_3)$  will be greater or less than the threshold when  $x(t_1) = \theta$ , the new measurement must be delayed such that the cost of a correct decision has substantially increased (due to the time-dependence of the payoff matrix), reducing the potential value of the information.

Figure 3.17 shows the expected value of new information  $V(x(t_1), t_2)$  as a function of the initial state  $x(t_1)$ , for several times  $t_2$  at which the new information

is measured. Information has zero value when  $t_2 = t_1$ , for all values of  $x(t_1)$ . This results from the assumption that the measurements are perfect, so that a second measurement at time  $t_1$  would provide no improvement in the knowledge about the state. Information also has zero value when  $t_2 = t_3$ , due to the time-dependence of the payoff matrix in this example.

When  $t_2$  is small (e.g.,  $t_2 = 0.1$ ), new measurements have small value because the long time between  $t_2$  and  $t_3$  prevents the measurement from substantially changing the model of  $x(t_3)$  (refer back to Figure 3.15). For intermediate values of  $t_2$  (e.g.,  $t_2 = 0.4$ ), information value increases substantially for intermediate values of  $x(t_1)$ , due to the improved ability to predict  $x(t_3)$  relative to the threshold. However, when  $x(t_1)$  is near the threshold, the measurement is still taken too early to allow confident prediction of whether  $x(t_3)$  will be greater or less than the threshold. When  $t_2$  is large (e.g.,  $t_2 > 0.6$ ), information value is largest for values of  $x(t_1)$  near the threshold, since the a priori uncertainty about the region in which  $x(t_3)$  will lie is largest and the measurement now allows a confident prediction of  $x(t_3)$ . For even larger values of  $t_2$  (e.g.,  $t_2 = 0.9$ ), information value decreases across all values of  $x(t_1)$ , because the cost of a correct decision increases as the decision is delayed.

The purpose of this example was to demonstrate the application of the information value model. The results suggest that the model is useful for understanding the time-dependence of information value. Chapters 4 and 5 will study two different examples as a means of extracting additional insight about the time-dependence of information value from the model. The following section discusses how to incorporate an imperfect measurement into the state model.



## Chapter 4

# Measurement and Communication of Airport Surface Conditions

### 4.1 Introduction

This chapter demonstrates the application of the time-dependent information value model presented in the Chapter 3, as an approach to determining when information should be measured/communicated. The chapter is not intended to be an exhaustive analysis of measurement and dissemination requirements for airport surface observations. Rather, the example is used to: (1) illustrate the formulation of the necessary models and the calculation of expected information value, and (2) illustrate general characteristics of information value time-dependence.

#### 4.1.1 Motivation

Historically, airport surface conditions have been manually observed, approximately hourly unless weather changes require an intermediate “special” observation.

Recently, the Automated Surface Observing System (ASOS) replaced manual observations at many airports, primarily to reduce operating costs at airport towers. ASOS measures the airport surface conditions every minute, because it lacks the ability (which the human observers possessed) to recognize when changes in the weather are significant to aircraft operations and require a special observation. The high ASOS measurement rate may also be a result of the technology being available and the additional cost over a lower measurement rate being small. Furthermore, because the automated sensors only see a small section of the sky, ASOS provides lower quality information than manual measurements. For example, consecutive ceiling measurements can vary dramatically if a single cloud, in an otherwise clear sky, passes over the narrow field-of-view ceiling sensor. ASOS's high measurement frequency partly compensates for the reduction in information quality, allowing pilots to construct a level of knowledge about the surface conditions similar to that which was previously provided by higher quality information, by filtering the multiple measurements.

Surface observations are disseminated to pilots either by a controller reading the data in response to a pilot request or by pilots listening to the continually broadcast Automatic Terminal Information Service (ATIS) recording. The ATIS message states the time at which the reported weather was observed and is updated whenever new measurements become available. Operating procedures require a pilot to listen to the ATIS recording before contacting the terminal area or tower controller and the controller to inform the pilot if the information has changed. A consequence of this procedural approach to information dissemination is that pilots en-route to an airport do not automatically receive special observations. Therefore,

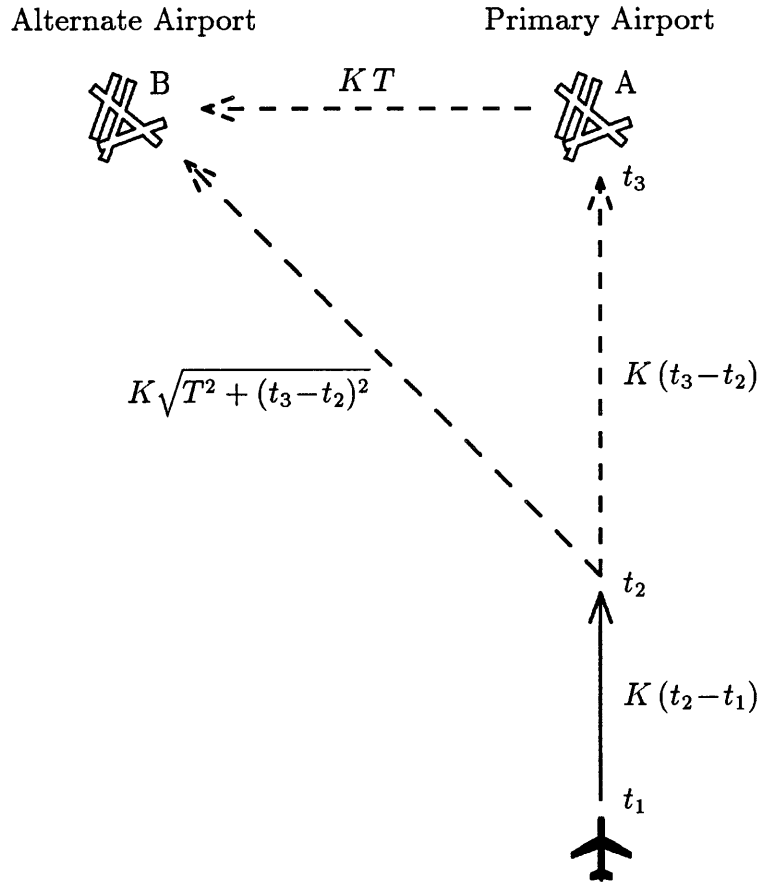
currently, a pilot must repeatedly query the ground in order to detect a change in the surface conditions. The pilot will use information about the surface conditions at the primary airport to decide whether or not to divert to an alternate airport, before arriving at the primary airport, reducing flight time and fuel consumption.

One user of airport surface measurements is a pilot who is en-route to an airport at which conditions are marginal for a non-precision instrument landing, when the aircraft or runway is not certified for category (CAT) I precision instrument operations. The FAA procedure for a non-precision instrument landing allows the aircraft to descend to a Minimum Descent Altitude (MDA) and fly along the extended runway centerline, using localizer guidance. If, by the Missed Approach Point (MAP), the ceiling and surface visibility do not allow the pilot to continue the landing using visual guidance, the pilot must perform a missed approach, either diverting to an alternate airport or waiting for conditions at that airport to improve. Therefore, in order for a non-precision instrument landing to be permitted, the ceiling (measured at the MAP at the time the aircraft reaches the MAP) must be greater than the MDA, and the surface visibility (measured from the MAP along the extended centerline) must be sufficient for the pilot to see the runway.

## 4.2 Problem Formulation

### 4.2.1 Geometry

Assume a pilot is en-route to airport A, where he had planned to land (Figure 4.1). At time  $t_1$ , when the aircraft was some distance from the destination, the pilot received measurements of the current ceiling  $x(t_1)$  and surface visibility



**Figure 4.1. Geometry of the example decision problem.** A pilot is en-route to airport A, where surface conditions at time  $t_1$  were marginal for a non-precision instrument landing. At time  $t_2$ , new surface observations are available, allowing the pilot to better predict the surface conditions at the arrival time  $t_3$ . The pilot must decide whether to continue to airport A or divert to airport B.

at airport A. The visibility reported at  $t_1$  was unlimited and was not expected to decrease significantly prior to the pilot's arrival, predicted to be at time  $t_3$  if he continues along his current flight plan. However, the ceiling at  $t_1$  was close to the 1000 foot Minimum Descent Altitude (MDA) for a non-precision instrument landing. Assume the aircraft equipment is not certified for a precision instrument

landing. Consequently, the pilot is uncertain whether, at the time of arrival, a non-precision instrument landing will be permitted or a missed approach will be required. The pilot has also received information about the surface conditions at airport B and is confident that they will be above that airport's MDA at all times at which he might arrive at that airport. Furthermore, the aircraft has sufficient fuel to first fly to airport A and then, if the conditions at the time of arrival do not permit landing, continue to airport B.

Assume that at time  $t_1$  the pilot had decided to continue toward airport A. At a later time  $t_2$  ( $t_1 \leq t_2 \leq t_3$ ) the pilot has the opportunity to receive a new ceiling measurement, allowing him to make a better estimate of the ceiling at the predicted arrival time  $t_3$ , and must again decide to either attempt to land at airport A or divert to the alternate airport B. The new measurement at time  $t_2$  has value because it improves the ability to model the ceiling at time  $t_3$  and, therefore, increases the probability the decision will be correct. Assume the measurements at  $t_1$  and  $t_2$  are accurate and current at those times. In general, the information may be inaccurate, due to measurement errors, or may already be old when it is received, due to a communication delay or the state being measured infrequently.

Figure 4.1 shows the geometry of the decision problem.  $T$  is the distance between airport A and B, measured in flying time.  $K$  is the per-unit-time cost of flying. The following values are assumed:  $T = 1$  hour,  $K = 150$  \$/hour, and  $t_3 - t_1 = 2$  hours. Let the Minimum Descent Altitude (MDA)  $\theta$  be 1000 feet. The proceduralized decision problem depends on a single state variable – the ceiling at airport A at time  $t_3$ . The MDA divides the state space into two regions, distinguished by whether a non-precision instrument landing is allowed or a missed

approach required.

To apply the method of Chapter 3 to this decision problem requires that we define the payoff matrix, model the pilot's decision, and model the ceiling at airport A. The following sections will present these models.

#### 4.2.2 Payoff Matrix

The pilot's decision at time  $t_2$  is either to continue to airport A or to divert to airport B. The possible outcomes are that the ceiling at airport A at time  $t_3$  is above or below the Minimum Descent Altitude (MDA). The payoff matrix  $\kappa$ , given in Table 4.1, summarizes the costs for each decision/outcome pair. The cost incurred from the pilot's decision depends on the total distance traveled, measured in terms of flight time. Although airport A is closer than airport B, flying directly to airport B is less expensive than first flying to airport A and then continuing on to airport B, because the ceiling at airport A is below minimum. Note that the minimum cost achievable through the decision depends on the ceiling at airport A at time  $t_3$ , which the pilot is not able to control. The elements of the payoff matrix also depend on the geometry of the airports and the aircraft at time  $t_1$ . The sensitivity of information value to the geometry will be discussed in Section 4.4. To model the pilot's preference to land at airport A, a constant cost  $\Lambda$  is assessed whenever the pilot decides to divert to airport B.

Assume that airport A will be closed to non-precision instrument landings at time  $t_3$  (i.e.,  $x(t_3) < \theta$ ). As the decision is delayed (i.e., as  $t_2$  increases), the cost  $\kappa_{22}$  of the correct decision (to divert to airport B) increases, while the cost  $\kappa_{21}$  of the incorrect decision (to continue to airport A) remains constant. Therefore, the

**Table 4.1. Payoff matrix for the airport surface observations example.**  $K$  is the per-unit-time cost of flying.  $T$  is the distance between airports A and B, measured in flying time.  $\Lambda$  models the pilot's preference to land at airport A.

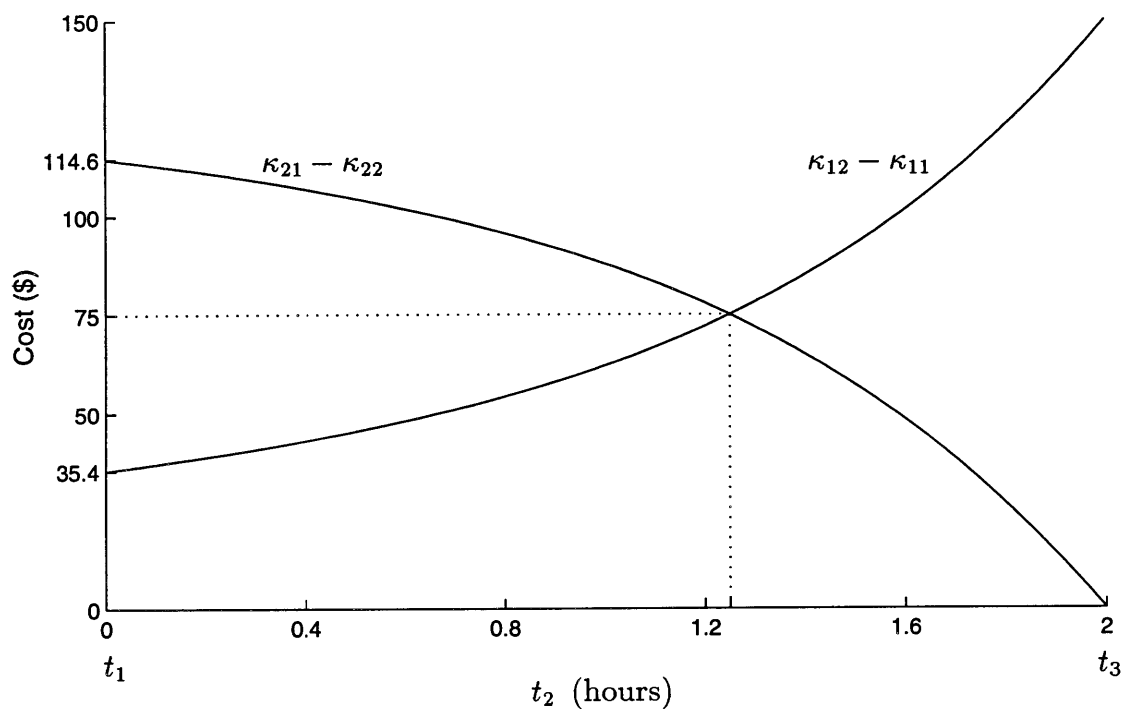
Ceiling at $t_3$ (Airport A)	Decision at $t_2$	
	Continue to Airport A	Divert to Airport B
Above MDA	$K(t_3 - t_1)$	$K(t_2 - t_1 + \sqrt{T^2 + (t_3 - t_2)^2}) + \Lambda$
Below MDA	$K(t_3 - t_1 + T)$	$K(t_2 - t_1 + \sqrt{T^2 + (t_3 - t_2)^2}) + \Lambda$

penalty  $\kappa_{21} - \kappa_{22}$  for the incorrect decision decreases as the decision is delayed, equaling zero when  $t_2 = t_3$ , plotted in Figure 4.2.

Alternatively, assume that the airport will be open to non-precision instrument landings (i.e.,  $x(t_3) > \theta$ ). Although the cost  $\kappa_{11}$  of the correct decision (to continue) is constant with respect to  $t_2$ , the cost  $\kappa_{12}$  of the incorrect decision (to divert) increases. Therefore, the penalty  $\kappa_{12} - \kappa_{11}$  for the incorrect decision increases as the decision is delayed. Also notice in Figure 4.2 that the sum of  $\kappa_{12} - \kappa_{11}$  and  $\kappa_{21} - \kappa_{22}$  is constant with respect to  $t_2$ .

#### 4.2.3 Modeling the Pilot's Decision

The pilot bases his decision to either continue to airport A or divert to airport B on his internal model of the probability that the ceiling at airport A will be



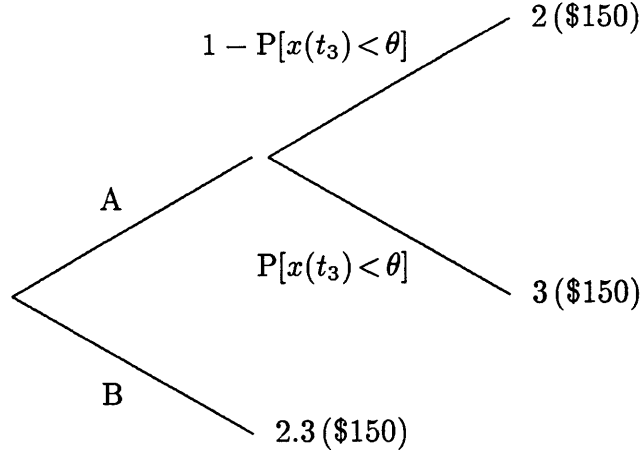
**Figure 4.2. Time-dependence of the penalties for incorrect decisions.**  $\kappa_{12} - \kappa_{11}$  is the penalty for unnecessarily diverting to airport B.  $\kappa_{21} - \kappa_{22}$  is the penalty for continuing to airport A when it will be closed.

above the threshold at time  $t_3$ , and his knowledge of the costs associated with each combination of the two possible decisions and the two possible outcomes of the state.

This example assumes the pilot makes his decision to minimize the expected cost. Although recent literature (Patrick, 1996) has suggested that many of the decisions which pilots make are not made according to a clearly defined utility function, this assumption remains useful in studying many aviation decision problems and information management issues.

Figure 4.3 draws the decision with which the pilot is confronted as a decision





**Figure 4.3. A decision tree model of the decision with which the pilot is confronted. For the case  $t_2 - t_1 = 0.5$  hours.**

tree, for the case  $t_2 - t_1 = 0.5$  hours. The costs of the three possible decision outcomes are found from the payoff matrix (Table 4.1). The minimum expected cost decision rule is to select A when  $3 P[x(t_3) < \theta] + 2 (1 - P[x(t_3) < \theta]) < 2.3$ , which is true for  $P[x(t_3) < \theta] < 0.3$ , and B otherwise.

Define  $C_A$  and  $C_B$  to be the expected costs for the pilot deciding to continue to airport A and divert to airport B, respectively. Notice that  $\kappa_{12} = \kappa_{22}$  for all  $t_2$ .

$$C_A(t_2) = P[x(t_3) < \theta | I] \kappa_{21}(t_2) + P[x(t_3) > \theta | I] \kappa_{11}(t_2) \quad (4.1a)$$

$$\begin{aligned} C_B(t_2) &= P[x(t_3) < \theta | I] \kappa_{22}(t_2) + P[x(t_3) > \theta | I] \kappa_{12}(t_2) \\ &= \kappa_{12}(t_2) \end{aligned} \quad (4.1b)$$

The minimum expected cost (MEC) decision rule is to fly to airport A when  $C_A < C_B$ , and to divert to airport B otherwise. Define  $p(t_2)$  to be the value of  $P[x(t_3) < \theta]$

for which  $C_A = C_B$ .

$$p \kappa_{21} + (1 - p) \kappa_{11} = p \kappa_{22} + (1 - p) \kappa_{12} = \kappa_{12} \quad (4.2)$$

$$\begin{aligned} p(t_2) &= \frac{\kappa_{12} - \kappa_{11}}{\kappa_{21} - \kappa_{11} - \kappa_{22} + \kappa_{12}} \\ &= \frac{\kappa_{12} - \kappa_{11}}{\kappa_{21} - \kappa_{11}} \end{aligned} \quad (4.3)$$

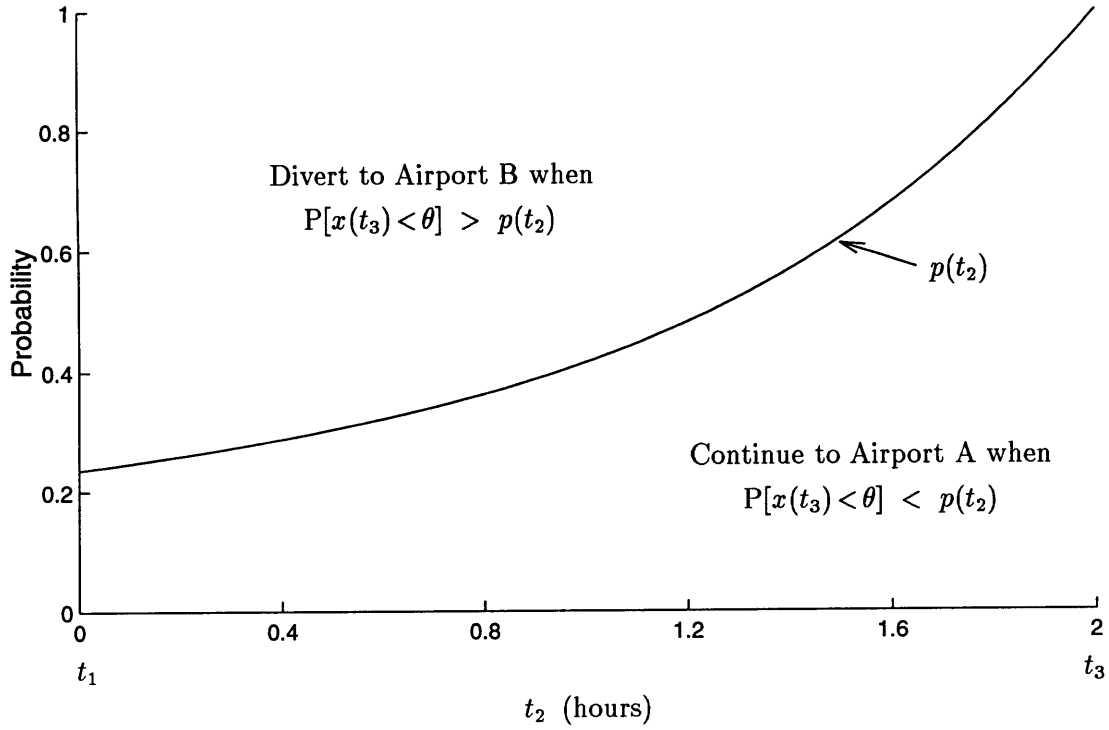
The minimum expected cost decision rule may be rewritten: fly to airport A when  $P[x(t_3) < \theta] < p(t_2)$ , and divert to airport B otherwise. Figure 4.4 plots  $p(t_2)$  versus  $t_2$ . When  $t_2$  is large,  $p(t_2)$  is large, because the distance to airport A is much smaller than the distance to airport B. Therefore, the probability that the ceiling at airport A will be below the threshold at  $t_3$  must be very large for diverting to airport B, without first trying to land at airport A, to minimize the expected cost.

To calculate the value of information, we require the probabilities that each action will be chosen. For the MEC decision rule, the probability of each action being chosen is either 1 or 0.

$$\text{If } P[x(t_3) < \theta | I] < p(t_2) \quad \text{then} \quad \begin{cases} P[a(t_2) = A | I] = 1 & \text{and} \\ P[a(t_2) = B | I] = 0, \end{cases} \quad (4.4a)$$

$$\text{otherwise} \quad \begin{cases} P[a(t_2) = A | I] = 0 & \text{and} \\ P[a(t_2) = B | I] = 1. \end{cases} \quad (4.4b)$$

Notice that the decision model depends on the available information, because the probability that the state will be below the threshold at time  $t_3$  is calculated from



**Figure 4.4.** The minimum expected cost decision policy. The minimum expected cost decision policy is to continue to airport A when  $P[x(t_3) < \theta] < p(t_2)$ , and to divert to airport B otherwise.

a model of the state which depends on the available information. Therefore, new information can change the pilot's decision.

In general, an agent other than the pilot, for example a weather-observer on the ground, may be valuing the information. If the information-valuing agent perceives the costs differently than the pilot, he may use a different payoff matrix to measure the value of information than the pilot uses to make the decision. The information-valuing agent may also have a different model of the airport ceiling than the pilot, due to additional information not available to the pilot or a different model of the state dynamics. These issues will be discussed in Section 4.4.

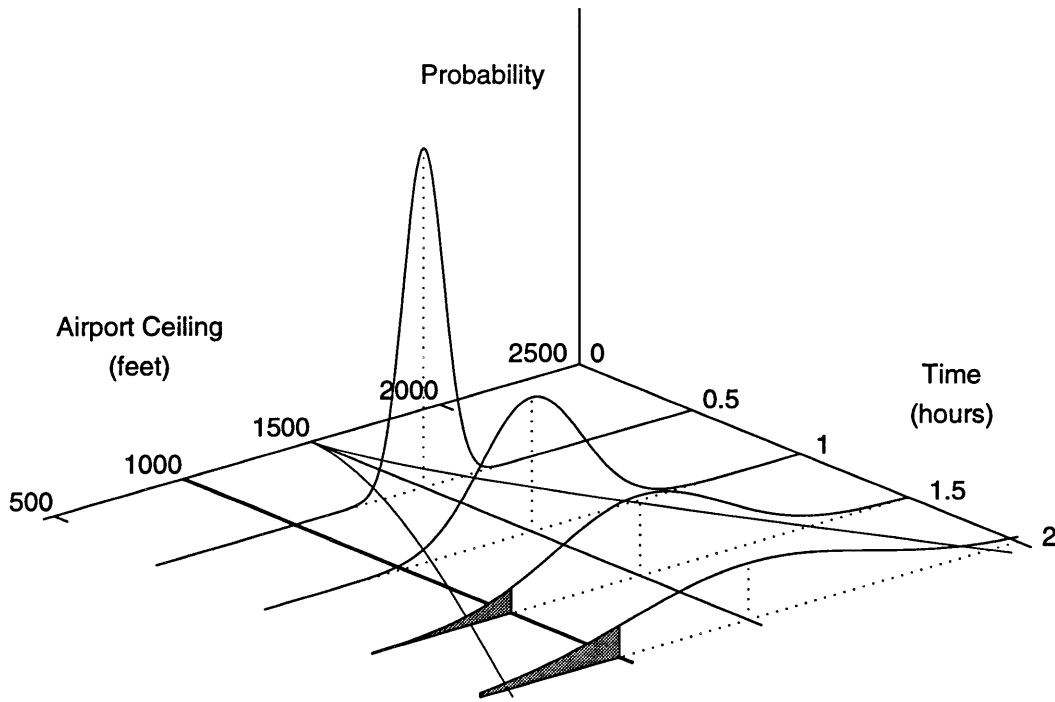


Figure 4.5. Example of state prediction, showing ceiling dynamics model.  $x(t_1) = 1500$  feet.

#### 4.2.4 Modeling the Airport Ceiling

The proceduralized decision whether or not a landing is allowed depends on a single state variable  $x$  – the ceiling at airport A – at time  $t_3$  – the predicted time of arrival at airport A. The state dynamics are modeled as an integrated, first-order Markov process, with time constant  $\beta = 1$  hour and root-mean-squared-value  $\sigma = 200$  feet. The parameter  $\sigma$  determines the rate at which the state may diverge from its expected value (i.e., the rate at which uncertainty grows). Appendix A presents this model. Figure 4.5 shows an example of the ceiling dynamics model, for  $x(t_1) = 1000$  feet. A Kalman filter provides the predicted models of the state variable at time  $t_3$ . Appendix A also describes the Kalman filter algorithm.

The model of the state dynamics predicts that the ceiling will most likely remain constant, but that the uncertainty in that estimate increases exponentially with the age of the prior measurement. Although this model is not appropriate for very long prediction times, because the unbounded variance eventually predicts a finite probability that the ceiling will be negative, it is useful when little is known about the trend in the ceiling (i.e., when a forecast is not available).

Recall that the purpose of this example is to exercise the information value model, rather than to provide a definitive solution to the question of how frequently airport surface observations should be updated. More realistic models of ceiling dynamics are available in the literature. For a sample of the work that has been done in this area, see Whiton and Berecek (1982), Berecek (1983), Henry and Wilson (1993), Chornoboy, Matlin, and Morgan (1994), Clark (1995), Willand and Boehm (1995), and Yu and Hocker (1995). Sheridan's (1970) approach – with increasing measurement age, the state model approaches a constant probability density function that has finite variance and is independent of the previous measurement (e.g., a PDF based on historical statistics about the airport surface conditions) – would be an appropriate model for very long prediction times.

### 4.3 Results

The following values for the parameters were assumed in the problem definition:  $t_3 - t_1 = 2$  hours,  $T = 1$  hour,  $K = 150$  \$/hour,  $\Lambda = \$0$ ,  $\beta = 1$  hour,  $\sigma = 200$  feet, and the Minimum Descent Altitude (MDA)  $\theta = 1000$  feet.

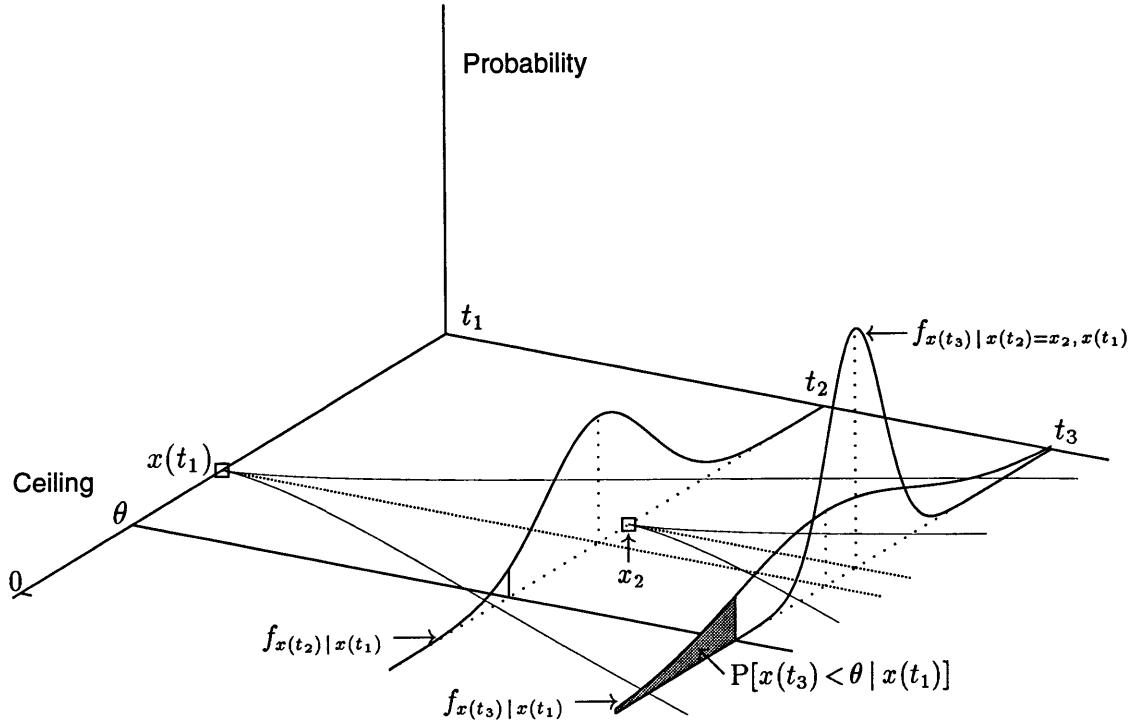
The expected value of information  $V(x(t_1), t_2)$ , repeated from (3.26), is a function of the decision time  $t_2$  and the initial ceiling  $x(t_1)$ . The expected uncertainty cost  $R|I$ , repeated from (3.8), depends on the available information  $I$ , which either is limited to the initial measurement  $x(t_1)$  or also includes the new measurement  $x_2$ .  $\kappa_{ij}$  is the element from row  $i$  and column  $j$  of the payoff matrix (Table 4.1). The decision model is given by (4.4). Figure 4.6 illustrates the three state models which enter into Equation (4.6): the a priori model of the state at time  $t_3$ , the a posteriori model given a particular value of  $x_2$ , and the distribution of possible values of  $x_2$  over which the integration is performed.

$$\begin{aligned} R|I = & P[a(t_2)=A|I] P[x(t_3)<\theta|I] (\kappa_{21} - \kappa_{22}) \\ & + P[a(t_2)=B|I] P[x(t_3)>\theta|I] (\kappa_{12} - \kappa_{11}) \end{aligned} \quad (4.5)$$

$$V(x(t_1), t_2) = \int_{x_2} f_{x(t_2)|x(t_1)}(x_2|x(t_1)) \left| R|x(t_1) - R|(x_2, x(t_1)) \right| dx_2 \quad (4.6)$$

Notice that the value of a new measurement at time  $t_2$  can only be measured relative to a baseline. In the current approach, value is defined as the change in the expected uncertainty cost, where the baseline is the expected uncertainty cost for the decision made at  $t_2$  with only the initial information. Other value metrics can be defined. For example, an alternative baseline against which to measure the impact of information is the expected uncertainty cost for the decision made at  $t_1$  with only the initial information.

To illustrate the process of calculating information value, Figure 4.7 plots several intermediate probabilities and the conditional expected uncertainty costs as



**Figure 4.6. Illustration of information value definition.** The state at time  $t_3$  is modeled by a probability density function, based on the available information and a stochastic model of the state dynamics. A new measurement taken at time  $t_2$ ,  $x_2$ , will improve the model of the state at time  $t_3$ . However, prior to receiving the measurement,  $x_2$  is not known.

functions of  $x_2$  (i.e., the new measurement that is received at time  $t_2$ ), for the case  $t_2 = 0.95$  hours and  $x(t_1) = 1200$  feet.

Figure 4.7a plots the a priori and a posteriori (i.e., with and without the new measurement, respectively) probabilities that the ceiling at airport A will be above the MDA at time  $t_3$ , as functions of the new measurement  $x_2$ . The a priori probability is constant. The larger the value of the new measurement  $x_2$ , the more likely the ceiling will be above the MDA at time  $t_3$ . It is also possible for the new measurement to be less than the original measurement, increasing the probability

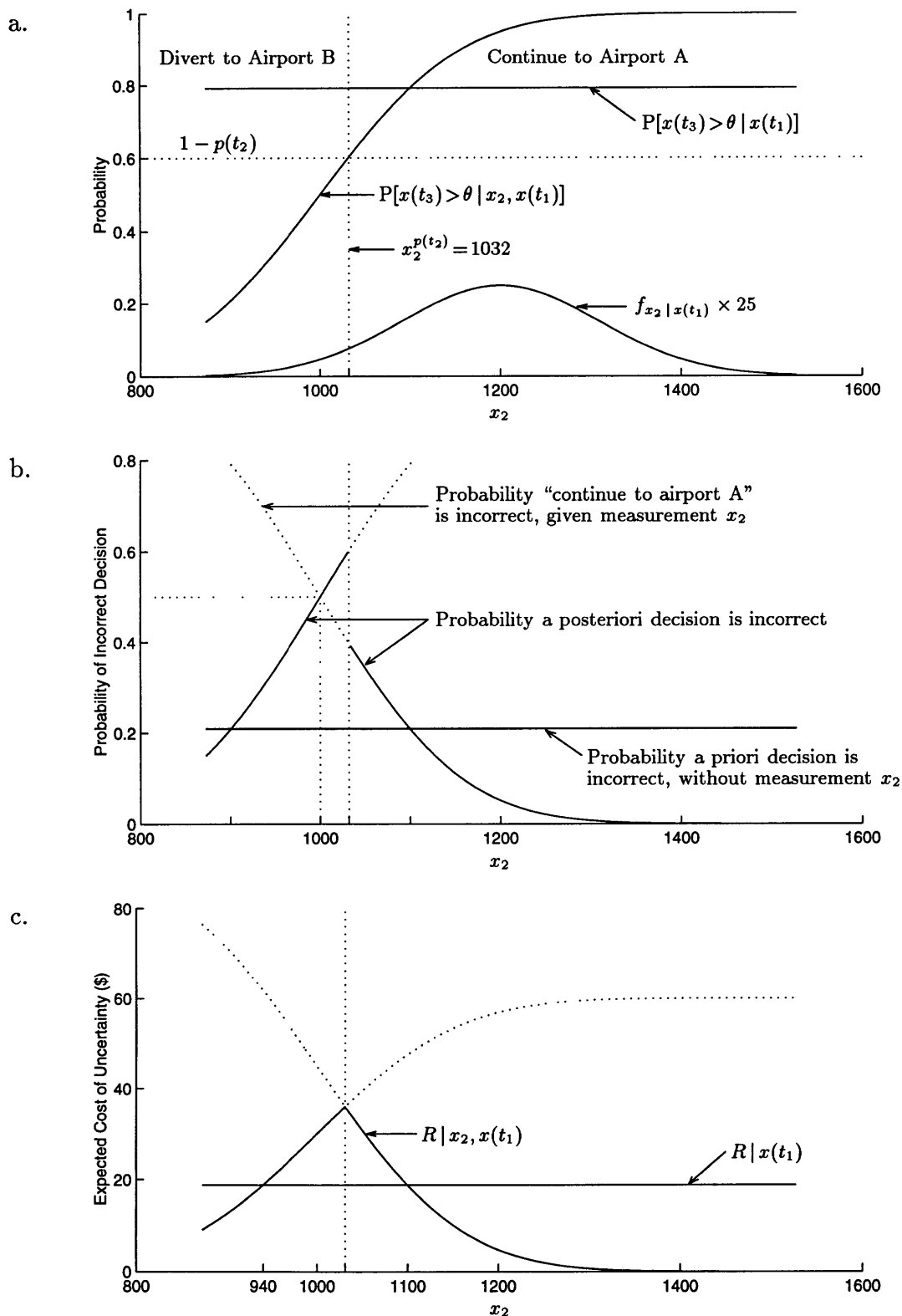
that the ceiling will be below the MDA at time  $t_3$ . Figure 4.7a also shows the probability density function  $f_{x_2|x(t_1)}$  for the new measurement taking on the value  $x_2$ , given the ceiling at time  $t_1$  is  $x(t_1)$  and assuming the previously described model of the state dynamics.

$p(t_2) = 0.4$ . The minimum expected cost (MEC) decision is to continue to airport A when  $P[x(t_3) < \theta] < p(t_2)$  (or, equivalently, when  $P[x(t_3) > \theta] > 1-p(t_2)$ ), and to divert to airport B otherwise. Table 4.2 summarizes the terminology that has been introduced earlier.

The a priori MEC decision – the decision, based only on the initial information, that minimizes the expected cost – is to continue to airport A. The a posteriori MEC decision – the decision, incorporating the new information, that minimizes the expected cost – depends on what measurement is received. Define  $x_2^{p(t_2)}$  to be the new measurement for which the expected costs for the two possible a posteriori decisions are equal (i.e., when  $x_2 = x_2^{p(t_2)}$ ,  $P[x(t_3) < \theta | x_2, x(t_1)] = p(t_2)$ , by definition). The a posteriori MEC decision is to continue to airport A when  $x_2 > x_2^{p(t_2)}$ , and to divert to airport B otherwise. Notice that when  $x_2 < x_2^{p(t_2)}$ , the new information causes the a posteriori decision to be different than the a priori decision. In this example,  $x_2^{p(t_2)} = 1032$  feet. That  $x_2^{p(t_2)}$  is greater than  $\theta$ , implies the decision to divert to airport B minimizes the expected cost even for some values of the new measurement that are greater than the threshold.

Figure 4.7b plots the probabilities that the a priori and a posteriori decisions are incorrect (i.e., will not minimize the expected cost). The probability that the decision to continue to airport A is incorrect equals the probability that the ceiling will be below the MDA at time  $t_3$ . Similarly, the probability that the decision to





**Figure 4.7. Calculation of information value.** For the case  $x(t_1) = 1200$  feet and  $t_2 = 0.95$  hours.

**Table 4.2. Terminology used in calculating information value.**

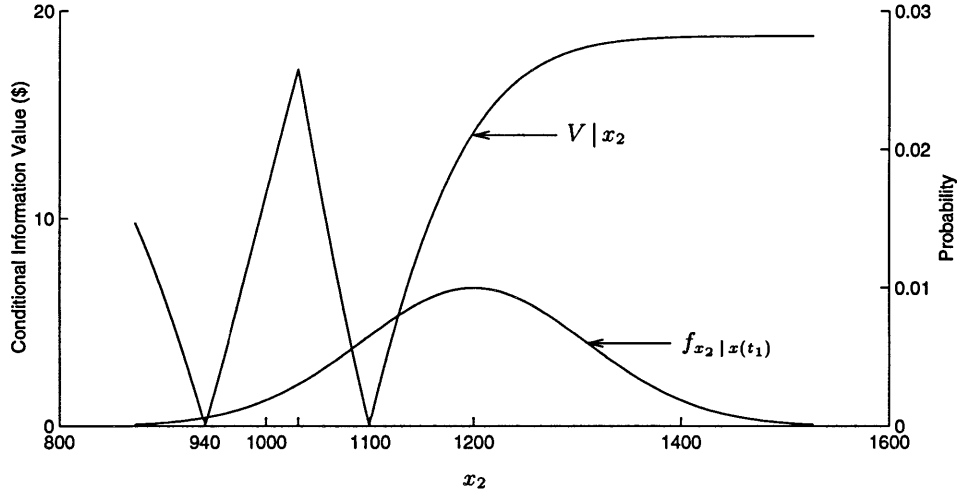
Symbol	Definition
$x(t)$	Ceiling at airport A at time $t$ .
$x_2$	The new measurement that is received at time $t_2$ .
$t_1$	Time at which the initial measurement is taken.
$t_2$	Time at which a new measurement is taken and the pilot must make the decision.
$t_3$	Time at which the aircraft will arrive at airport A.
$\theta$	Minimum Descent Altitude (MDA), defined by the procedure.
$P[x(t_3) > \theta   I]$	The probability that the state at time $t_3$ , $x(t_3)$ , will be greater than the MDA $\theta$ , calculated from the model of the state that is derived from the information $I$ , $f_{x(t_3) I}$ .
$f_{x(t) I}$	Model (probability density function) for the state at time $t$ , based on information $I$ .
$I$	The available information, either limited to the initial measurement $x(t_1)$ or also including the new measurement $x_2$ .
$p(t_2)$	The value of $P[x(t_3) < \theta]$ for which the expected costs of continuing to airport A or diverting to airport B are equal.
$R   I$	Expected uncertainty cost, calculated from the model of the state that is derived from the information $I$ .
$V   x_2$	Conditional information value. The change in the expected uncertainty cost, if the new measurement is $x_2$ .

divert to airport B is incorrect equals the probability that the ceiling will be above the MDA at time  $t_3$ . Both the a posteriori decision and the probability that it is incorrect are functions of the new measurement  $x_2$ . Although the expected costs for the two possible a posteriori decisions are equal at  $x_2 = x_2^{p(t_2)}$ , the probabilities that the decisions are incorrect are unequal, because the payoff matrix is not symmetric.

The probability that a decision is incorrect is calculated using a model of the state. The probability that the a posteriori decision is incorrect is calculated using

the a posteriori state model (i.e., the state model that incorporates the measurement  $x_2$ ). The probability that the a priori decision is incorrect is calculated using the a priori state model (i.e., the state model based only on the original measurement). The probability that the a priori decision is incorrect might, alternatively, be calculated using the a posteriori state model. However, whereas the first approach positively values information that changes the confidence in the a priori decision, this approach would only attribute value to the new measurement when the a posteriori decision is different from the a priori decision.

Figure 4.7c plots the expected uncertainty costs before and after receiving the new measurement ( $R|x(t_1)$  and  $R|x_2, x(t_1)$ , respectively) as functions of the new measurement. The a priori expected uncertainty cost is a constant. Although a new measurement greater than 1100 feet does not change the decision, it reduces the probability that the decision is incorrect and, consequently, reduces the expected uncertainty cost. The decision is also unchanged when  $1032 < x_2 < 1100$ . However, the resulting a posteriori probability that the decision is incorrect is larger than the a priori probability, because the a priori model of the state at time  $t_2$ ,  $f_{x(t_2)|x(t_1)}$ , considered this measurement unlikely. Although measurements in this range increase the expected uncertainty cost, they are valued positively because they reveal that the region in which the state will lie at time  $t_3$  cannot be predicted with as much confidence as had been thought, and that this larger uncertainty should be recognized when making the decision. The increase in expected uncertainty cost also suggests that an additional future measurement may have value. When the measurement is less than 1032 feet, the a posteriori MEC decision is different than the a priori decision. When the new measurement equals  $x_2^{p(t_2)}$ , the a posteriori



**Figure 4.8. Conditional (on the new measurement) information value as a function of the possible measurements.** For the case  $x(t_1) = 1200$  feet and  $t_2 = 0.95$  hours.

expected uncertainty costs for the two possible decisions are equal. In fact, the a posteriori expected uncertainty cost is maximized by this measurement.

Figure 4.8 shows the conditional value of information  $V | x_2$  (i.e., the value of information given a specific measurement is received) as a function of the feasible measurements. The probability of each measurement occurring is also shown.

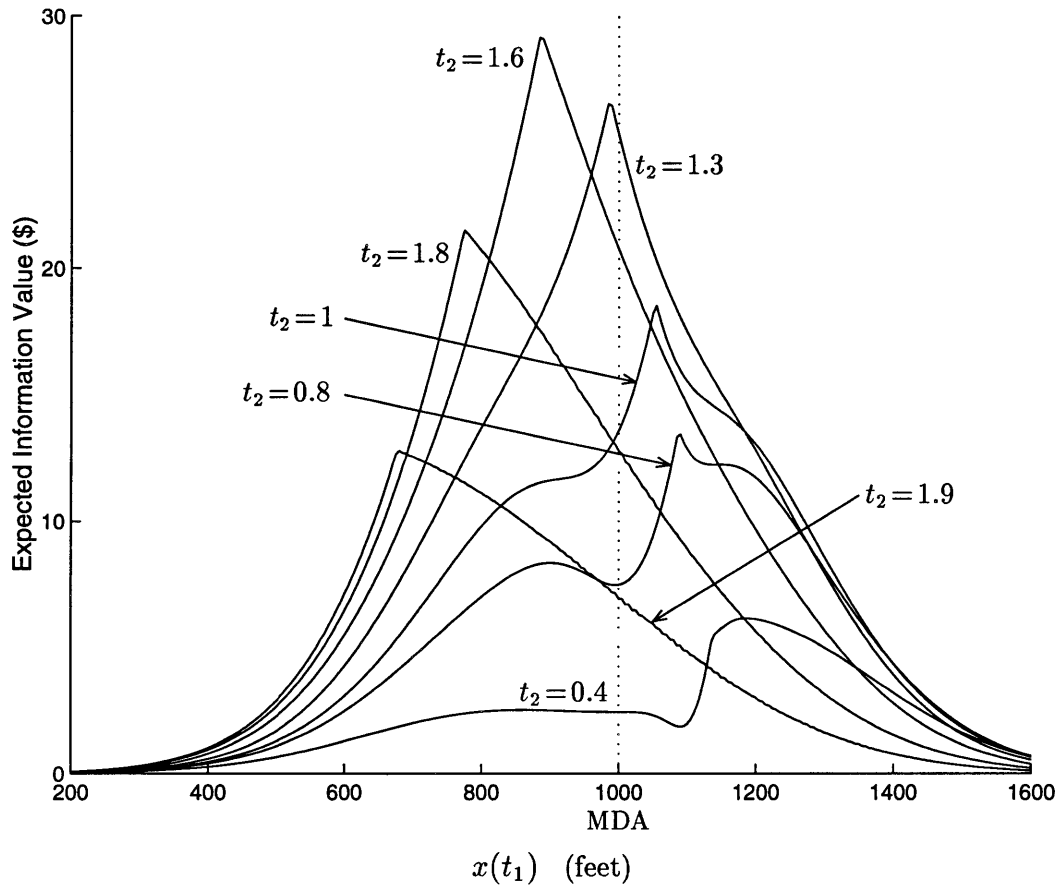
$$V | x_2 = \left| R | x(t_1) - R | (x(t_2), x(t_1)) \right| \quad (4.7)$$

Notice that if the new measurement is between 940 feet and 1100 feet, the a posteriori expected uncertainty cost is greater than the a priori expected uncertainty cost. However, these measurements are given positive value because they reveal an error or unjustified confidence in the a priori state model, and suggest that additional information should be sought. The conditional information value is zero when the a posteriori expected uncertainty cost equals the a priori expected uncertainty cost.

Expected information value equals the integral, over the feasible values of  $x_2$  (i.e., the measurements that have non-zero probability of occurring), of the conditional information value, weighted by the probabilities of those measurements occurring. The value of taking a new measurement at  $t_2 = 0.95$  hours when  $x(t_1) = 1200$  feet, the case discussed in Figures 4.7 and 4.8, is \$12.8.

Figure 4.9 plots the expected information value  $V(x(t_1), t_2)$  as a function of the initial ceiling  $x(t_1)$ , parametric in the time  $t_2$  at which the new measurement is taken. Although the model of the ceiling at time  $t_3$  may be more accurate when the new measurement is taken closer to time  $t_3$ , the benefit of increasing the probability that the decision is correct decreases as the decision is delayed, due to the time-dependence of the payoff matrix. Therefore, a tradeoff exists in selecting  $t_2$ . The new measurement achieves less of an improvement in the state model (i.e., the probability that the decision will be correct) when taken at an earlier time, compared to a later time. However, the benefit of making a correct decision (i.e., the difference between the costs of the correct and incorrect decisions) is larger when the new measurement is taken at an earlier time. This tradeoff was discussed in Chapter 3.  $t_2 = 1.6$  hours provides high expected information value over all possible values of  $x(t_1)$ .

In Figure 4.9, the large change in expected information value for values of  $t_2$  less than 1 hour, that occurs over a small range of the initial ceiling  $x(t_1)$ , results because the a priori decision switches at a value of  $x(t_1)$  in this range, and the a priori expected uncertainty costs for the two decisions are substantially different, while the a posteriori expected uncertainty cost, which is averaged over the possible measurements, does not change substantially across this range of  $x(t_1)$ .



**Figure 4.9.** Expected information value (\$), plotted as a function of the initial measurement  $x(t_1)$ , for several values of the time  $t_2$  at which the information is measured. The Minimum Descent Altitude,  $\theta$ , is 1000 feet.

Figure 4.10 plots the expected information value  $V(x(t_1), t_2)$  as a function of the time  $t_2$  at which the new measurement is taken, parametric in the initial ceiling  $x(t_1)$ . When the initial ceiling is near or below the 1000 foot MDA, the value of the new measurement is largest for values of  $t_2$  near  $t_3$ . Assume aircraft is near airport A (i.e.,  $t_2$  is close to  $t_3$ ). The penalty for unnecessarily diverting to airport B is very large, while the penalty for continuing to airport A when airport A will be closed is small. When the ceiling at time  $t_1$  is 900 feet, for example, the a priori

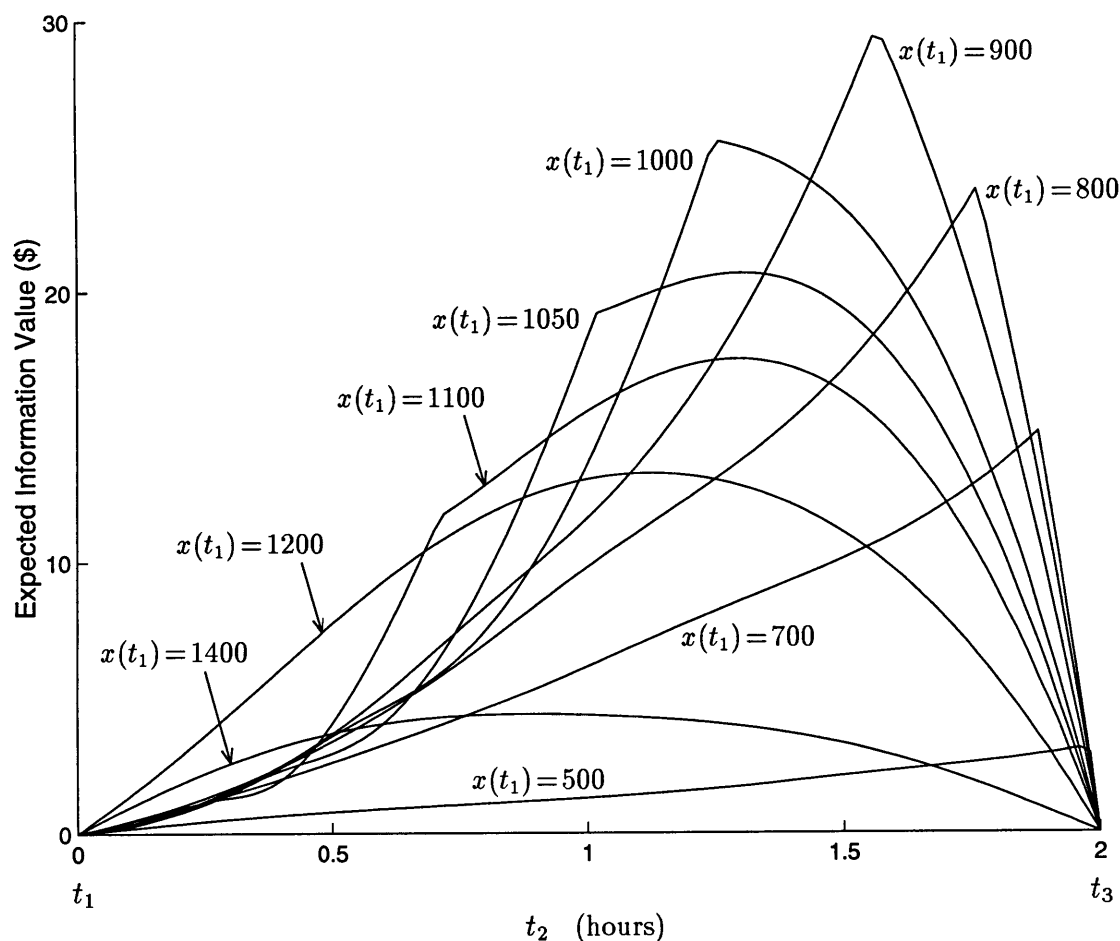


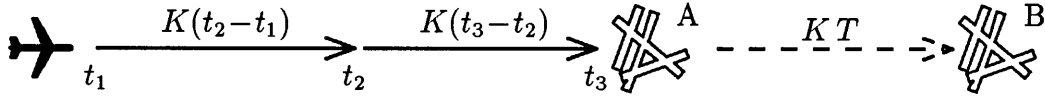
Figure 4.10. Expected information value (\$), plotted as a function of the time  $t_2$  at which the information is measured, for several values of the initial measurement  $x(t_1)$ . MDA = 1000 feet

MEC decision is to divert to airport B, even though there is a large probability that the ceiling will be above the MDA at time  $t_3$  and, therefore, the decision will be incorrect. Therefore, the a priori expected uncertainty cost is very large. The a posteriori decision is likely to be correct, because the new measurement is taken close to time  $t_3$ . Therefore, when the new measurement reveals that the ceiling will be greater than the MDA at time  $t_3$ , the a posteriori decision will be to continue to airport A, avoiding the high penalty of unnecessarily diverting at a late time.

## 4.4 Discussion

### 4.4.1 Sensitivity to Problem Geometry

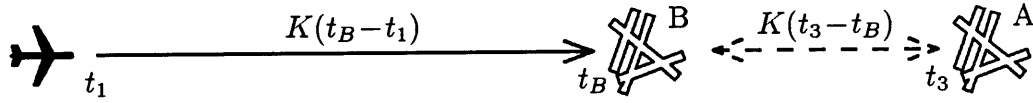
$x(t_1)$  and  $t_2$  are only two dimensions of the context of the decision problem. The other dimensions, related to the problem geometry and captured by the payoff matrix, were held constant in the previous results. If the decision problem was changed, for example, such that the shortest distance to airport B takes the aircraft over airport A, shown in Figure 4.11, then at no time  $t_2$  would a new measurement of the ceiling at airport A have any value, since it would not affect the pilot's decision to proceed toward airport A. This result would be reflected in the payoff matrix for this decision problem.



**Figure 4.11. An alternative airport geometry.** A new measurement taken at time  $t_2$  has zero value, for  $t_2$  between  $t_1$  and  $t_3$ .

Alternatively, if airports A and B were swapped (with respect to Figure 4.11), such that the pilot must fly over airport B on his way to airport A (Figure 4.12), then information would have zero value until the pilot reaches airport B. When  $t_2 = t_B$ , a new measurement would have large value – if the pilot chooses to fly to airport A when the ceiling at  $t_3$  will be below the MDA, he must then return all the way to airport B at a high cost. Information value decreases to zero as  $t_2$





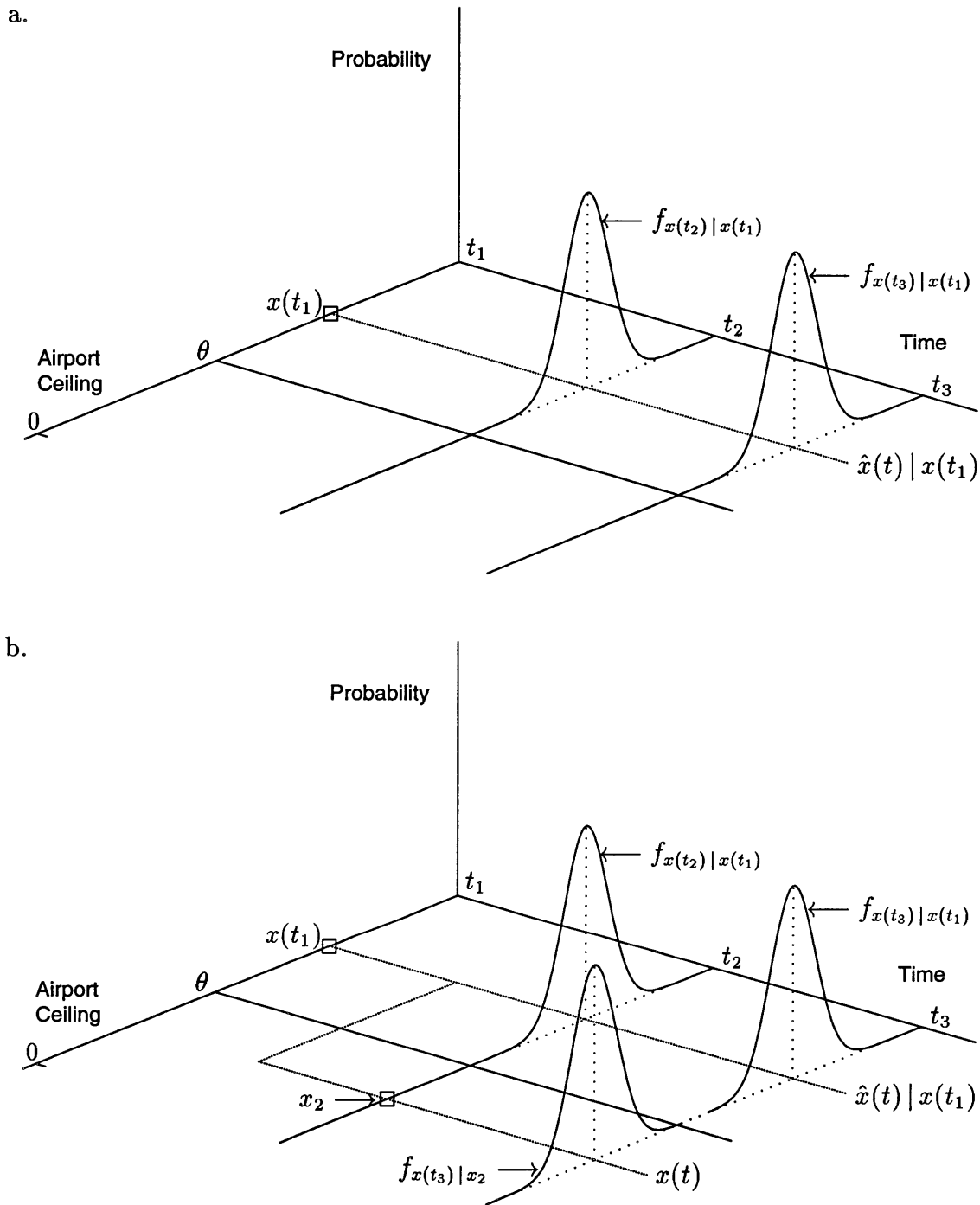
**Figure 4.12. An alternative airport geometry.** The value of a new measurement taken at time  $t_2$  is zero when  $t_2 < t_B$ , and decreases from a maximum when  $t_2 = t_B$  to zero when  $t_2 = t_3$ .

approaches  $t_3$  because the extra cost of continuing the remainder of the way to A, if the aircraft must return to airport B, (over turning around at time  $t_2$ ) decreases.

#### 4.4.2 Model Uncertainty

Consider the situation depicted in Figure 4.13a. Based on the initial information, a pilot is confident that the airport ceiling will be above the MDA at time  $t_3$ . Furthermore, the pilot does not believe a measurement at  $t_2$  that would allow the state to be below the threshold at  $t_3$  is possible. Therefore, the value of new information at  $t_2$ , measured from the perspective of the pilot, is zero.

What if the pilot's model of the future state is incorrect and the state follows a trajectory not considered feasible by the pilot? Such a situation might occur if, for example, the pilot is unaware of a weather front moving toward the airport, shown in Figure 4.13b. In this case, receiving a measurement at time  $t_2$  would reveal to the pilot the error in his state model, significantly changing his model for the state at time  $t_3$ . Although this measurement at  $t_2$  would have a high value to the pilot, the pilot does not know to request the new information.



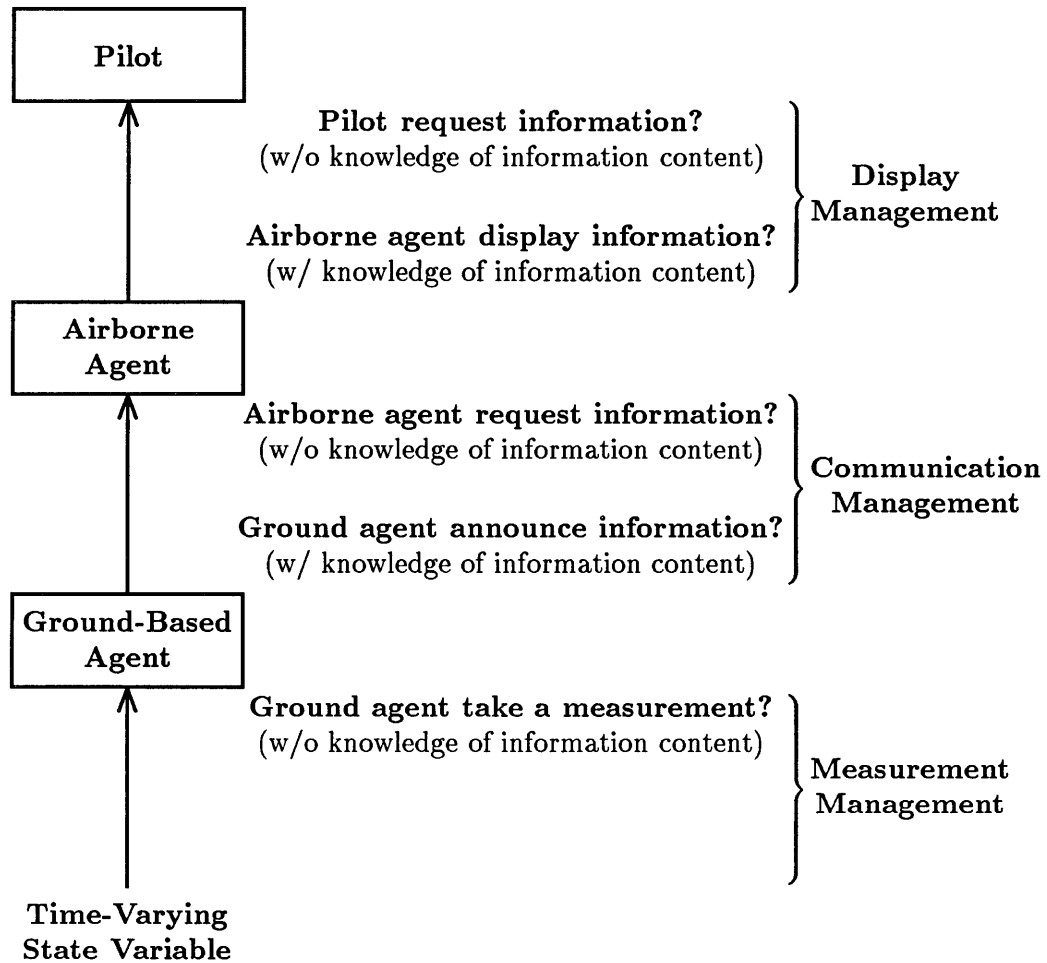
**Figure 4.13. The effect of an incorrect model on information value.** If the state is outside the range considered feasible by the decision-making agent's state model, a new measurement would change that model and, therefore, may have high value. However, the decision-making agent does not know to request the information.

Thus far, information value has been defined from the perspective of an *decision making agent* who does not know the content of the information. In many information management problems, responsibility for controlling information dissemination is shared between the *decision making agent*, that receives the information, and an *information-measuring agent*, who knows the content of the information. In general, both of these agents may be either human or automation. The following two sections discuss architectural issues relating to the participation of these two agents in managing information flow.

#### **4.4.3 Cooperative versus Non-Cooperative Information Dissemination Management**

Figure 4.14 illustrates a general architecture for managing the delivery of ground-measured information to support pilot decision-making. The architecture includes an autonomous agent between the pilot and ground-based agent whose purpose is to manage an on-board information cache. The bandwidth is much higher between the pilot and the airborne agent than between the airborne agent and the ground agent. Therefore, the pilot can access information cached on-board the aircraft much more quickly than information that must be retrieved from the ground.

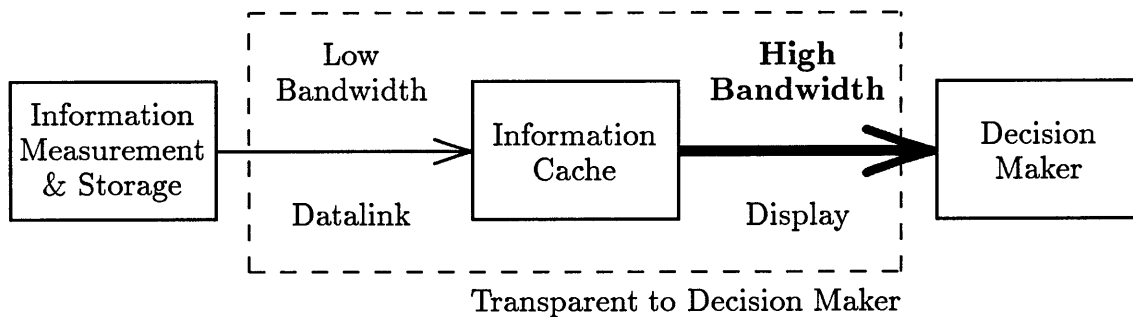
The architecture separates the problem of managing the flow of ground-sensed information to a pilot into three information management processes: measurement, air-ground communication, and display. The previously presented approach to valuing information may be used to study each of these information management processes.



**Figure 4.14.** General architecture for managing the delivery of ground-measured information to support pilot decision making.

A *cooperative* information management environment is one in which agents at both the measurement and decision-making ends of the information transfer can contribute to the information management process. For example, cooperative air-ground communication consists of both requests for information from the airborne-agent, which prior to receiving the information does not know what the content of

the information will be, and announcements of information from the ground-based agent, which can use knowledge of the information content in deciding when to announce the information. In a *non-cooperative* environment, the airborne-agent must initiate all information transfer. Similarly, this situation is repeated in the management of the display. The airborne automation, which possesses knowledge of the information content, must decide whether to forcibly display new information. The pilot must decide whether to request new information, without the benefit of knowing the information content. No opportunity exists for cooperation in the management of the measurement process. Therefore, the ground-based agent must decide when to record measurements, before knowing what the measurements will be.



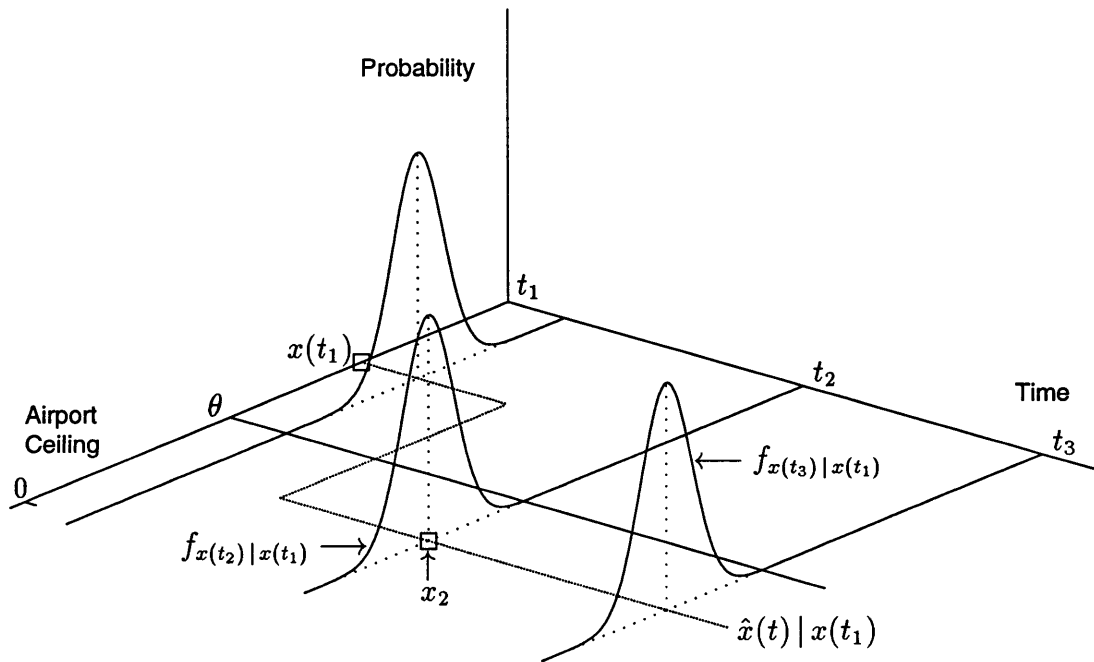
**Figure 4.15. Virtual bandwidth datalink.**

Frequently, the bandwidths available in the communication and display processes are not equal. The performance of interest is the delay in displaying required information. The display of locally-stored information to the decision-making agent is across a high bandwidth channel (e.g., a local-area network or computer bus).

By smartly managing the transfer of information across the low bandwidth ground-air datalink and locally caching information, the airborne autonomous agent can increase the effective bandwidth between the information source and the decision-making agent. By anticipating the decision-making agent's information requirements, this architecture, shown in Figure 4.15, can reduce the delay in delivering information to the decision-making agent. Consequently, the decision-making agent perceives a higher bandwidth datalink. This improvement in information throughput has been called *virtual bandwidth*. The technique smoothes peak demands on datalink bandwidth by incurring some of the “delay” in receiving the information across the datalink before the information is required. However, if the information is rapidly changing in time, in order for the cached information to be current when the decision-making agent requests it, the agent must either retrieve the current information continually or be able to accurately predict when the information will be required. The former approach would be an inefficient use of the low bandwidth datalink because most of the retrieved information would never be used.

#### 4.4.4 Valuing Information with Knowledge of its Content

When the content of the new information is known by the agent measuring the information's value, the agent's model (probability density function) for the the measurement ( $f_{x(t_2)|x(t_1)}$ ) is a unit impulse function at  $x_2 = x(t_2)$ , and zero elsewhere. The integral over the possible measurements in the expression for the expected value of information, Equation (4.6), is easily evaluated, and the expected information value equals the conditional information value given the new measurement, defined in Equation (4.7).



**Figure 4.16.** The effect of the decision-making agent's model of the state variable dynamics on information value. In contrast to Figure 4.13, if the pilot knows that a front is approaching and the airport ceiling will drop, then a measurement at  $t_2$  would have no value to the pilot.

Although the information-measuring agent knows the content of the new information prior to measuring its value, the agent does not know the decision-making agent's model of the state. Figures 4.13b and 4.16 illustrate the dependence of information value on the pilot's state model. In contrast to the example of Section 4.4.2, if the pilot knows a weather front is approaching the airport and expects the ceiling to drop below the threshold (i.e., the pilot's model for the future state is correct), a new measurement taken at time  $t_2$  will not change the pilot's state model and, therefore, will have no value to the pilot (Figure 4.16).

Therefore, to measure the value that new information would have to the

decision-making agent, the information-measuring agent requires a model of the decision-making agent's model of the state. If the information-measuring agent's model of the pilot's state model is incorrect – if the the agent assumes the pilot expects the weather front when he does not, for example – the agent will either over or under-value the information. To accommodate uncertainty relative to the pilot's state model, the information-measuring agent's model of the pilot's state model should be conservative from the perspective of valuing information. Furthermore, the model should be intuitive to the decision-making agent, so that he is able to predict under what conditions the information-measuring agent will automatically provide information. Two possible assumptions for the pilot's model are that the state will remain constant and that the state will follow a forecast which the pilot is known to have.

#### 4.4.5 The Value of Forecast Information

Assume that, based on information measured at time  $t_1$ , a weather front is forecast to move across an airport at some time between times  $t_a$  and  $t_b$ . Upon the front's arrival, the airport will close to non-precision instrument operations. Assume that the speed at which the front is advancing towards the airport is not known accurately. With no additional knowledge, the front's arrival time is assumed to be uniformly distributed between  $t_a$  and  $t_b$ . Figure 4.17a illustrates this scenario.

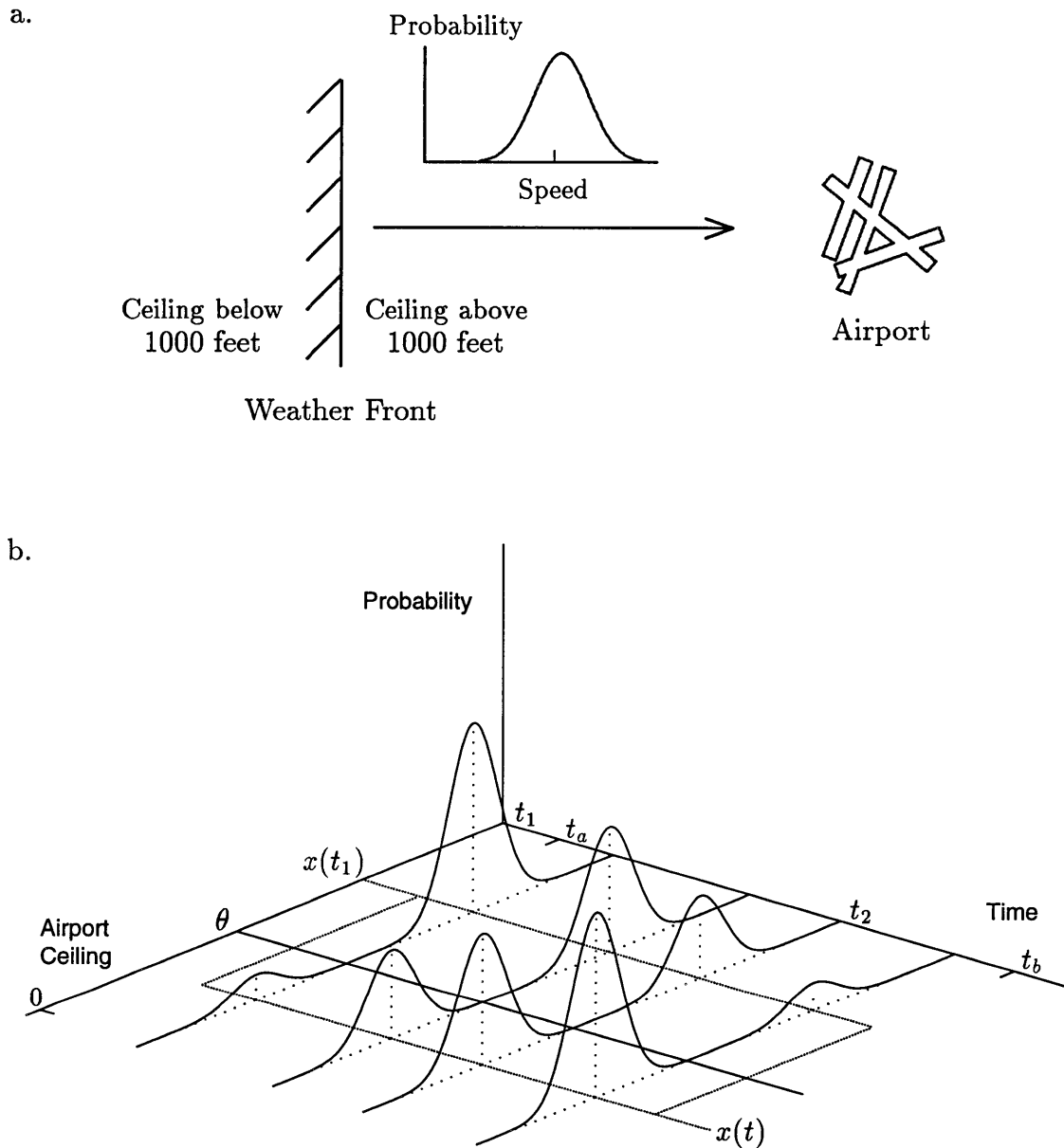
The random process modeling how the airport ceiling will evolve in time is piece-wise stationary. The expected values of the ceiling before and after the front reaches the airport are constants, above and below the MDA, respectively. The uncertainty in predicting the ceiling at a future time results from know knowing



which of these two feasible models of the random process is correct at that time. Figure 4.17b depicts models of the airport ceiling at four points in time, based on the information measured at  $t_1$ . Notice that the small variations around these two feasible expected values are not significant to a pilot deciding whether or not to divert because he will not be able to land. However, the uncertainty in whether or not the front will arrive at the airport prior to the aircraft is significant to his decision.

In a non-cooperative environment, for the pilot to know when the front arrives requires that he repeatedly query the ground-agent for a current ceiling measurement, until the front arrives. In a cooperative environment, the ground-agent promises to notify the pilot when the front arrives, providing the same knowledge about the front's arrival more efficiently. However, in both cooperative and non-cooperative architectures, measurements of the ceiling at the airport prior to the arrival of the front, do not improve the pilot's ability to predict whether or not he will arrive at the airport before the front.

A new measurement of the airport's ceiling at any time prior to time  $t_a$  will not improve the ability to predict the time at which the front will arrive. However, information that reveals the location and speed of the front (e.g., ceiling measurements at other nearby airports at several times that bracket the front passing those airports) would allow a more accurate forecast of the front's arrival time. In general, forecast information consists of measurements of state variables on which the proceduralized decision problem does not explicitly depend, but which enable a more accurate prediction of the relevant state variables. Chapter 5 will further study the value of forecast information (also referred to as derivative or intent information).



**Figure 4.17. Uncertainty in the time at which a weather front will reach the airport.** Part *a*: a weather front is forecast to move across an airport between times  $t_a$  and  $t_b$ . With no additional information, the arrival time is assumed to be uniformly distributed over this range. Part *b*: models of the airport ceiling at four times, based on information measured at time  $t_1$ .

## Chapter 5

# Dependent Aircraft Surveillance

### 5.1 Introduction

This chapter presents two examples in which the time-dependent information value model (Chapter 3) is applied to the problem of determining when – how frequently or at what specific times – aircraft should broadcast state measurements or intent information, in a dependent aircraft surveillance environment.

The example in Chapter 4 considered the expected value of only a single new measurement. Section 5.2 studies the expected value of multiple new measurements, in the context of predicting whether or not a conflict will occur between aircraft on intersecting trajectories, so that a resolution, if necessary, may be initiated at an early time. The time-dependent information value model is demonstrated as a tool for determining how many new measurements should be taken, and at what times.

Like the example in Chapter 4, this example assumes that the proceduralized decision problem depends on the values of the relevant state variables at a single,

well defined point in time –  $t_3$ . Section 5.3 relaxes this requirement, applying the model to identify the optimal, constant update rate for a continuous proceduralized decision problem. A scenario in which two aircraft are flying along parallel tracks is used, where the optimal update rate is a tradeoff between the cost of information and the expected cost of uncertainty.

Section 5.4 discusses the role of derivative and intent information in modeling the future motion of an aircraft and detecting when the aircraft deviates from that model. Section 5.5 revisits the paradigm of cooperative versus non-cooperative information management architectures in the context of dependent aircraft surveillance.

### 5.1.1 Motivation

Datalink-based dependent aircraft surveillance – aircraft reporting state measurements or intent information to controllers or other aircraft via datalink, to augment or replace radar surveillance – has been proposed as an enabling technology for reducing separation requirements and increasing operational flexibility (i.e., free flight) in both domestic and oceanic airspace.

For example, to achieve a low probability of a collision in oceanic airspace, historical limitations in aircraft surveillance and controllability by a central air traffic control (ATC) facility have led to a rigid route structure and restrictive operating procedures. This *track* system prevents aircraft from operating at the speeds and altitudes that are individually most efficient. To improve operating efficiency, airlines would like aircraft to be able to change their speeds and altitudes as they burn fuel, and to change their routes to take advantage of current weather and wind

information. Recent improvements in air-ground communication bandwidth and reliability, brought about by the advent of satellite-based datalink communication, will allow air traffic controllers to receive more frequent information about aircraft, with which to monitor aircraft separations, enabling separation requirements to be reduced and operational flexibility to be increased.

However, there is a cost for providing this information. Satellite-based datalink communication currently costs on the order of \$10 per transmission. Although the direct cost for using terrestrial-based datalinks is less than that for satellite-based datalinks, the larger number of aircraft requires that the limited datalink bandwidth be used efficiently. This constraint on the available bandwidth may be interpreted as a cost, similar to the direct cost for using a satellite-based datalink. Therefore, datalink applications should be carefully designed to minimize the number of messages required to provide operational benefits – i.e., how frequently necessary information must be updated.

## **5.2 Aircraft on Crossing Tracks**

To this point, this thesis has considered the value of a single new piece of information. This section will generalize the definition of expected information value to accommodate multiple new measurements.

### **5.2.1 Problem Statement**

Consider two aircraft flying along crossing trajectories, as shown in Figure 5.1. The objective is to study the value of new measurements to the problem of predict-

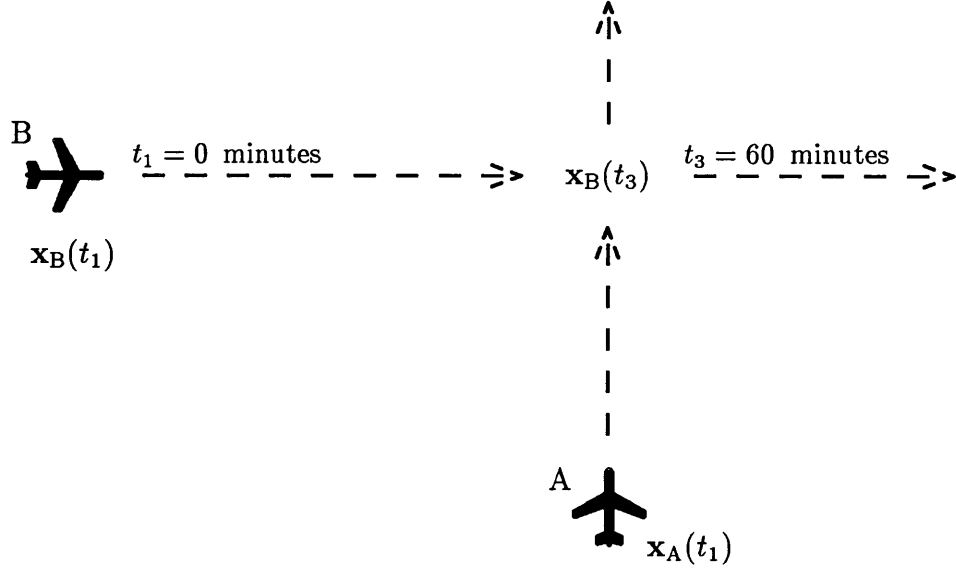
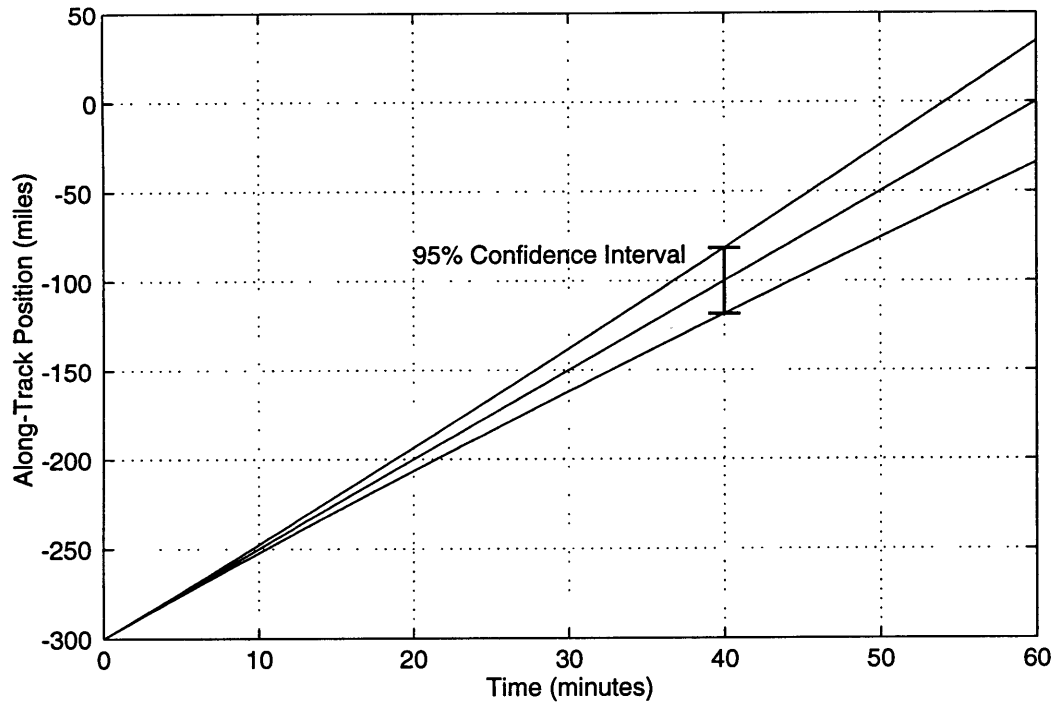


Figure 5.1. Two aircraft on crossing trajectories.

ing whether or not a conflict will occur. In this example, a conflict occurs if the separation between the aircraft is less than 5 miles at any time during the encounter.

To simplify the computations, assume the aircraft will closely follow their intended trajectories (i.e., will not deviate laterally or in altitude). At time  $t_1$ , aircraft A's position is measured, with some expected error. Its speed is modeled as an integrated, first-order Markov process (i.e., position is a twice-integrated Markov process). Figure 5.2 illustrates the rate at which uncertainty in the position of aircraft A grows with the time since the last measurement. Also assume that the speed of aircraft B is constant throughout the encounter and the position and speed are known at time  $t_1$ .

Several methods for calculating the probability that a conflict will occur, given an encounter geometry and a description of the uncertainties about the aircraft trajectories, are available in the literature. As an alternative to the Monte Carlo

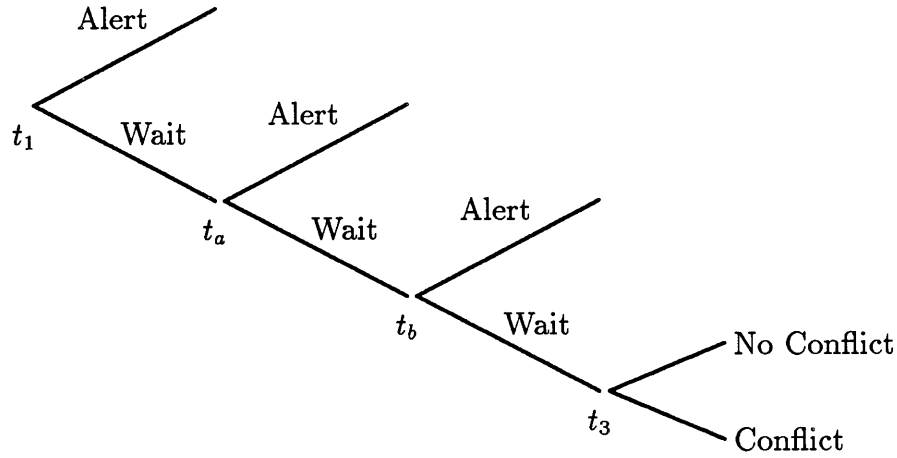


**Figure 5.2. Illustration of the state dynamics model for the position of aircraft A.**

approach used by Yang and Kuchar (1997), Krozel, Peters, and Hunter (1997) offer an analytic approach that will be used here. This method is explained in Appendix A.

### 5.2.2 Collective Expected Information Value of Multiple Measurements

The information value model introduced in Chapter 3, Equation (3.26) and Figure 3.11, may be generalized to calculate the collective expected information value of multiple new measurements. This will be shown for two measurements; extension to more than two measurements should be apparent.



**Figure 5.3. Decision tree for the decision problem at time  $t_1$ .**  
New measurements are available at times  $t_a$  and  $t_b$ .

Rather than a single new measurement being taken at a time  $t_2$ , assume that two new measurements will be taken at times  $t_a$  and  $t_b$ . The decision tree for the decision problem with which the decision-making agent is confronted at time  $t_1$  is shown in Figure 5.3. Since information is not available continuously, the decision to either continue monitoring or intervene is made at the discrete times at which new information is received ( $t_1$ ,  $t_2$ , and  $t_b$ ).

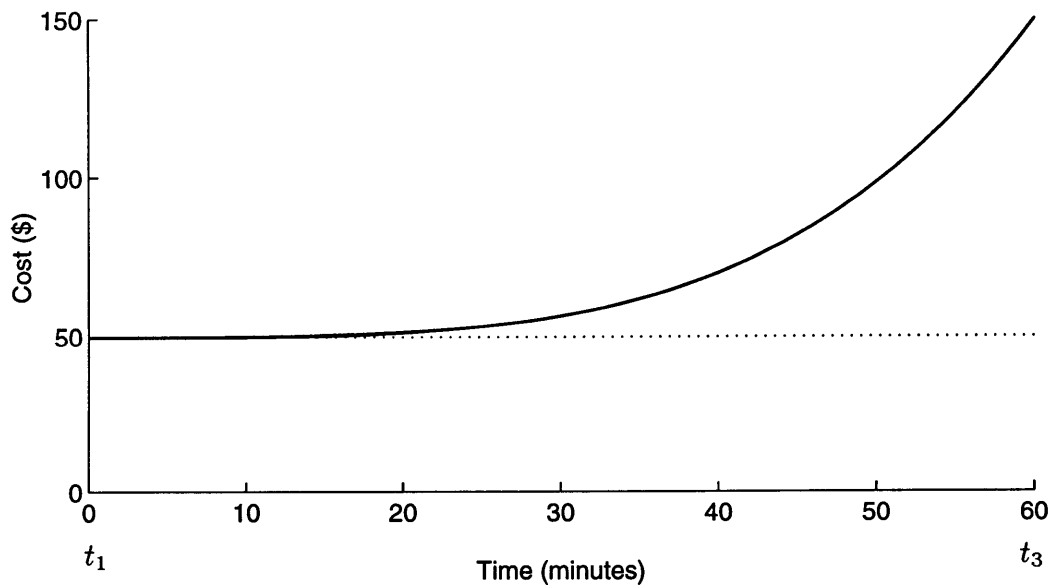
The payoff matrix for the decision at a time  $t$  is given in Table 5.1. The cost for alerting equals the cost  $K(t)$  for an avoidance maneuver initiated at that time. The cost for not alerting equals  $K(t_3)$  if a conflict occurs, and \$0 otherwise. The cost  $K(t)$  for an avoidance maneuver initiated at time  $t$ , plotted in Figure 5.4, increases exponentially as the time remaining until the conflict decreases.

The collective expected value of two measurements taken at times  $t_a$  and  $t_b$  equals the expectation over the possible measurements at time  $t_a$  of the values of those measurements, plus the expected value of the measurement at time  $t_b$ , given



**Table 5.1. The payoff matrix for the decision made at time  $t$ .**  
 $K(t)$  is the cost of an avoidance maneuver initiated at time  $t$ , shown in Figure 5.4.

	Decision at time $t$	
	No Alert	Alert
No Conflict	0	$K(t)$
Conflict	$K(t_3)$	$K(t)$



**Figure 5.4. Cost  $K(t)$  for an avoidance maneuver initiated at time  $t$ .**

**Table 5.2. Terminology used in Equation (5.1).**

Symbol	Definition
$V(x(t_1), t_a, t_b)$	The expected value of information, given the initial measurement is $x(t_1)$ and new measurements are taken at times $t_a$ and $t_b$ .
$f_{x(t_a) x(t_1)}$	The probability density that the measurement at time $t_a$ will be $x_a$ , given the initial measurement was $x(t_1)$ .
$f_{x(t_b) x_a, x(t_1)}$	The probability density that the measurement at time $t_b$ will be $x_b$ , given the measurement at time $t_a$ was $x_a$ and the initial measurement was $x(t_1)$ .
$R x_t$	The expected uncertainty cost for the decision made at time $t$ with information $x_t$ .
$x_t$	The position of aircraft A at time $t$ .

the measurement received at time  $t_a$ . Table 5.2 explains each of the terms in the Equation (5.1).

$$\begin{aligned}
 V(x(t_1), t_a, t_b) = \int_{x_a} f_{x(t_a)|x(t_1)} \left[ \left| R|x(t_1) - R|x_a \right| \right. \\
 \left. + \int_{x_b} f_{x(t_b)|x_a} \left| R|x_a - R|x_b \right| dx_b \right] dx_a \quad (5.1)
 \end{aligned}$$

Notice that Equation (5.1) calculates the collective value that is expected prior to receiving either of the new measurements. After the first new measurement is received, the expected value of the second measurement may change. Figures 5.5 and 5.6 illustrate the dependence of the expected information value of the second measurement on the content of the first measurement. Figure 5.5 shows the effect of expected miss distance on the expected value of new information. Figure 5.6

shows the effect of the time at which the previous measurement was taken on the expected value of new information.

To determine when to take the two measurements, Equation (5.1) may be evaluated over all possible combinations of  $t_a$  and  $t_b$ . The measurements should be taken at the times which maximize the collective expected value. Figure 5.7 shows the collective expected information value as a function of the times at which two new measurements are taken. Equation (5.1) may be generalized to calculate the collective expected value of more than two measurements. The appropriate number of new measurements can be determined by calculating the net value for one, two, three, and so on, new measurements, where *net value* is defined as the expected information value minus the information cost.

### 5.2.3 Results

Figure 5.8 shows the optimal times at which to take 1, 2, 3, or 4 new measurements; Figure 5.9 shows the cumulative expected information value for measurements taken at those times. If a single new measurement is taken, its expected value is largest when it is taken at 48 minutes. Taking the measurement at this time is the best compromise between the ability to make the correct decision, and the benefit for making the correct decision at an early time.

If two new measurements are taken, the collective expected information value is largest when one measurement is taken at 22 minutes and the other at 52 minutes. Notice from Figure 5.6 and Figure 5.7 that the expected value of taking a measurement at 52 minutes after having taken a measurement at 22 minutes is approximately equal to the expected value of a measurement taken at 52 minutes

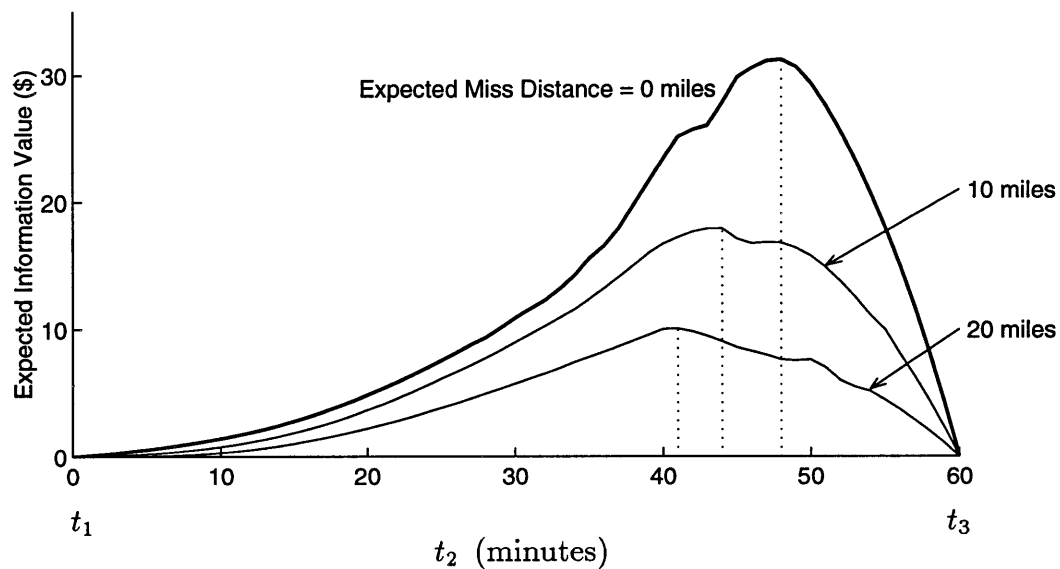


Figure 5.5. Expected Information Value of a single new measurement taken at time  $t_2$ , for three different expected miss distances.

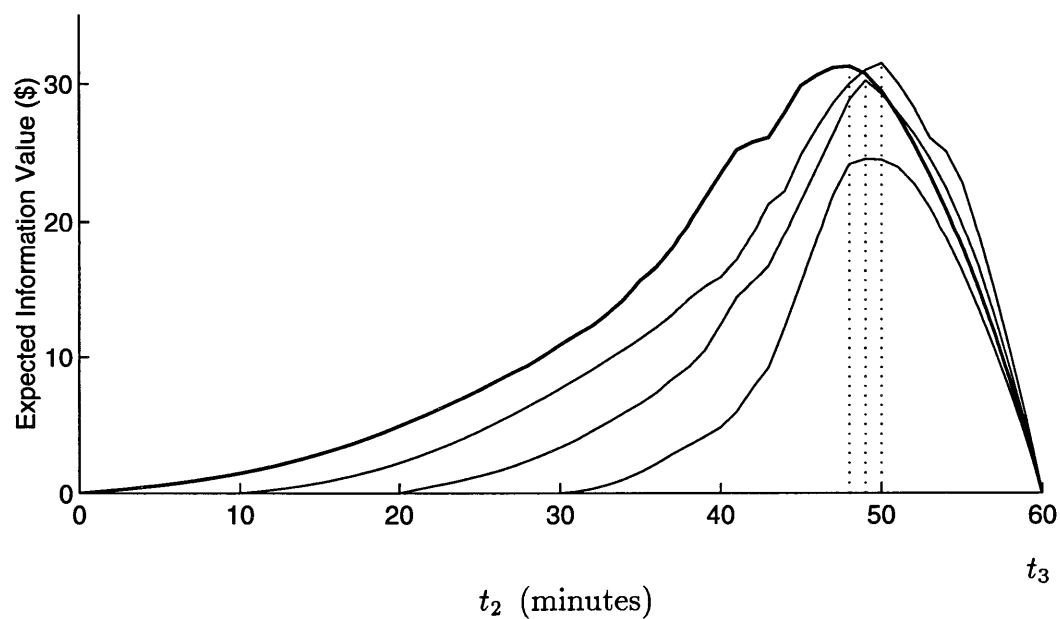
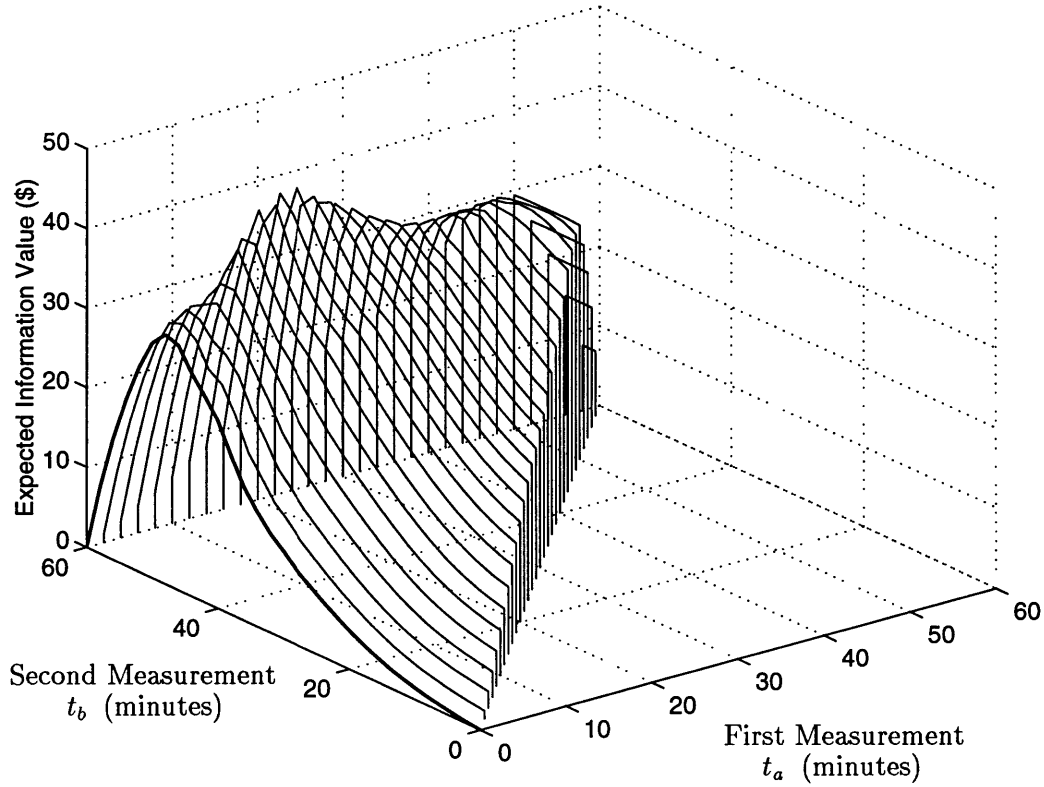


Figure 5.6. Expected Information Value for a single new measurement taken at time  $t_2$ , for four different values of  $t_1$ . The expected miss distance is 0 miles.



**Figure 5.7. Expected Information Value for two new measurements.** The expected miss distance is 0 miles,  $t_1 = 0$  minutes, and  $t_3 = 60$  minutes.

when no earlier measurement is taken. The expected value of the early measurement results because, prior to receiving the measurement, there is some chance that the measurement will allow the correct decision to be made at a very early time. If the correct decision is to alert, the cost of the avoidance maneuver is less than if the maneuver were initiated at a later time. If the correct decision is to not alert, the cost of the second measurement may be avoided.

For 3 and 4 new measurements between times  $t_1$  and  $t_3$ , the intervals between the optimal measurement times decrease as the time remaining until the possible

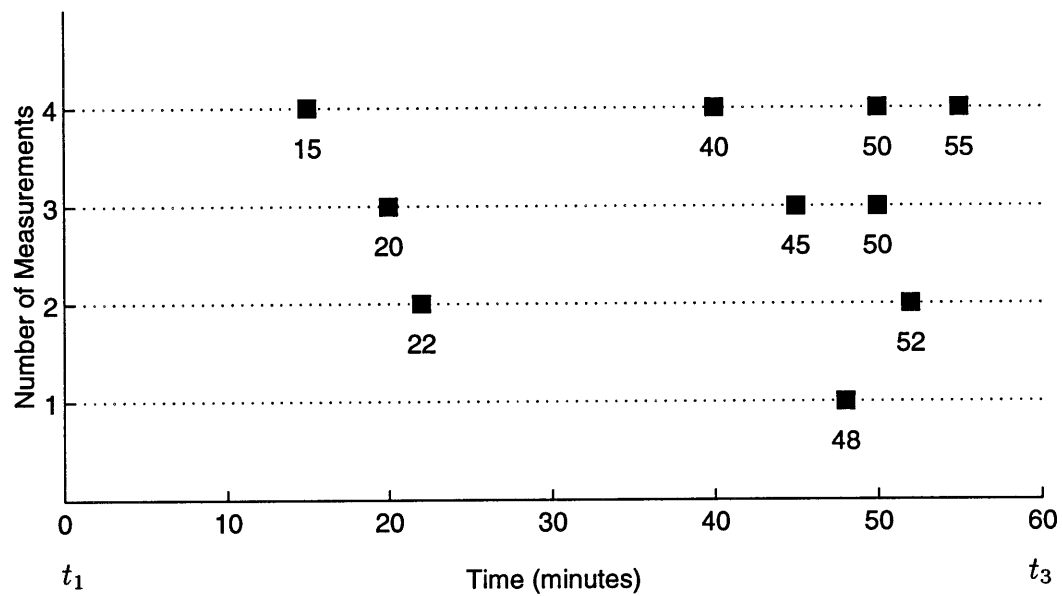


Figure 5.8. Optimal times at which to take 1, 2, 3, or 4 new measurements. Expected miss distance is 0 miles.

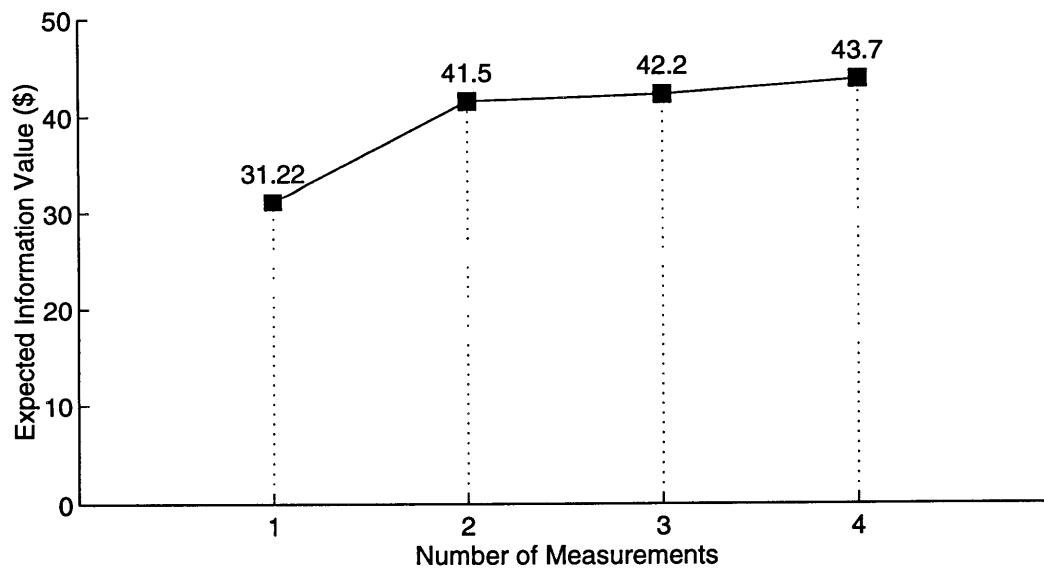


Figure 5.9. Maximum Expected Information Values achievable from 1, 2, 3, and 4 new measurements. Expectation taken at time  $t_1$ , prior to receiving any of the new measurements.

conflict decreases.

Figure 5.9 exhibits diminishing returns in the collective expected information value, as the number of measurements is increased. An additional measurement should be sought only if the resulting increase in the collective information value is greater than the cost of the measurement. In this way, the approach may be used to determine how many new measurements should be taken, in addition to when those measurements should be taken.

Note that the expectations underlying the results in Figures 5.8 and 5.9 are calculated prior to receiving any of the new measurements (i.e., at time  $t_1$ ). After the first new measurement is received, the expected values of the future measurements, and the times at which future measurements have maximum expected value, may change. Given the received information, the plan for what information should be sought in the future may be re-calculated using the same method. The resulting computational demand makes the approach more applicable as an off-line analysis tool, than as an on-line algorithm (i.e., programmed to run in real-time in aircraft avionics).

#### 5.2.4 Summary

The model of time-dependent information value (Chapter 3) was generalized to calculate the collective expected information value of multiple new measurements. Given the per-measurement cost of information, the model may be used to determine how many measurements should be taken, and at what times.

An encounter between two aircraft on intersecting trajectories was studied. The time intervals between optimally spaced measurements decrease as the time

remaining until the possible conflict decreases. The cumulative information value, expected prior to receiving any of the measurements, exhibits diminishing returns with an increasing number of measurements. If the cost of information is \$5/measurement, two measurements should be taken at 22 minutes and 52 minutes, based on the information available at time  $t_1$ . However, the measurement that is received at 22 minutes may change the expected value of future information, so that, given the new knowledge, taking one additional measurement at 52 minutes may not be optimal. The same method may be used to re-plan how many and when future measurements should be taken.

The computational demand of the calculation and the need to re-plan when to take future measurements each time a measurement is received make the approach best suited as an off-line analysis/design tool.

## 5.3 Aircraft on Parallel Tracks

### 5.3.1 Problem Statement

This section applies the time-dependent information value model to determine the optimal tradeoff between the cost of information and the cost of uncertainty, when measurements are taken periodically.

Assume that two aircraft A and B are flying at equal speeds on parallel oceanic tracks, as shown in Figure 5.10. Let  $\mathbf{x}_A(t)$  and  $\mathbf{x}_B(t)$  represent their positions, and  $d(t)$  the lateral separation, at time  $t$ . The decision problem with which the controller is confronted is to monitor the aircraft separation and intervene if a situation which would otherwise result in a conflict arises. At every point in time,



the controller will either intervene or continue monitoring. A conflict is considered to occur when  $d < \theta$ , where  $\theta$ , the required separation, is 10 nautical miles in this example.

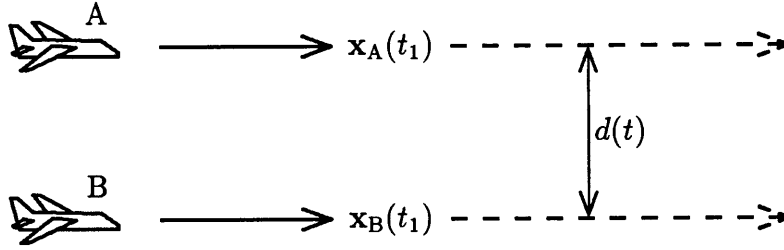


Figure 5.10. Aircraft on parallel tracks.

In this example, the possibility of a conflict is not isolated to a short period of time, as in the previous example, but may occur at any time. Therefore, the controller must continually monitor the positions of the aircraft, and continually decide whether or not intervention is required. To support monitoring or continuous control tasks of this type, information must be updated periodically. The optimal update interval is constant, because the context is constant as long as the aircraft remain on their nominal trajectories. If one of the aircraft deviates from its nominal path, the new encounter resembles the problem in Section 5.2. The objective of periodic taking periodic measurements is to detect an aircraft blunder. The update rate is chosen to balance the cost of latency in the detection against the cost of measurements.

### 5.3.2 Information Cost

If there is zero cost for seeking or using information – e.g., there is no direct cost for taking or receiving a measurement, the available bandwidth is not limited, there is no penalty for delaying the decision or control action to seek additional information, and the decision maker has unlimited time to process information – then information should be updated continuously. When there is a cost for seeking or using information, the information management problem is to balance the benefit of additional information against that cost.

Note that the assumption that only a single measurement may be taken between the initial measurement at time  $t_1$  and the critical decision time  $t_3$ , represents an implicit cost for seeking information. Including a direct information cost would not have changed the time-dependence of the information's value, assuming the cost is independent of the time  $t_2$  at which the measurement is taken. The effect of including a direct cost for the measurement would be to lower the net value of information equally across all values of  $t_2$ . The new measurement should be forgone only if the direct cost for the information exceeds the value of the information for all values of  $t_2$ .

Let  $\Delta t$  be the time between subsequent measurements. In the present problem, the context remains constant since the two aircraft are flying along nominally parallel trajectories. Therefore, the periodic measurement interval  $\Delta t$  is assumed to be constant. The goal of this section is to identify and study factors which affect the selection of the periodic measurement interval. In general, the optimal periodic measurement rate depends on the instantaneous context of the decision problem, which is continually changing. For example, when aircraft on crossing trajectories

are far apart, frequent updates have little value to the task of monitoring their separation. However, when the aircraft are near the intersection of their trajectories, frequent updates may have high value. Therefore, the measurement interval should be adjusted as the context changes. It is for this reason that much of the thesis considers the time-dependence of a single new measurement.

The total cost which accumulates over an interval  $\Delta t$ ,  $C_T^{\Delta t}$ , consists of the cost of the information received during the interval,  $C_I^{\Delta t}$ , and the *cumulative expected uncertainty cost* for the the interval,  $R^{\Delta t}$ .

$$C_T^{\Delta t} = C_I^{\Delta t} + R^{\Delta t} \quad (5.2)$$

Superscripts are used to indicate the period of time over which the cost has accumulated:  $\Delta t$  denotes a cost which has accumulated over a single measurement interval, and  $dt$  will denote the instantaneous rate at which cost accumulates. Subscripts are used to identify each cost: T denotes the total cost, and I denotes the cost of seeking information. Note that the cost of information must be measured in the same units as the cost of uncertainty (i.e., the units of the payoff matrix).

$C_I^{\Delta t}$  is the cost for a single measurement.  $C_I^{\Delta t}$  is typically a constant, although information of different “quality” (e.g., resolution or the number of derivatives that are measured) may be available at different costs. Define  $C_I^{dt}(\Delta t)$  to be the average cost of information per unit of time, within a measurement interval of length  $\Delta t$ .

$$C_I^{dt}(\Delta t) = \frac{C_I^{\Delta t}}{\Delta t} \quad (5.3)$$

### 5.3.3 Expected Uncertainty Cost When the Decision Task is Continuous

The *cumulative expected uncertainty cost* which accumulates over a single interval  $\Delta t$ ,  $R^{\Delta t}$ , is calculated next. Since the decision task is continuous, at every moment in time, the controller must decide whether to intervene or continue monitoring. The controller's model of the relevant states only changes when a new measurement is received. Therefore, if upon receiving a measurement the controller decides to continue monitoring, the controller will not intervene before the next measurement. A characteristic of monitoring decision problems is that the decision maker implicitly makes a default decision – to continue monitoring – whenever he does not explicitly take an action.

The row of the payoff matrix which corresponds to the decision to continue monitoring has two columns (i.e., the state will be in one of two possible regions): a conflict does not exist between the two aircraft, or a conflict does exist. Let the rates at which cost accrues for these decision outcomes be:  $\kappa_{11}$  equals \$0/minute and  $\kappa_{12}$  equals \$50/minute, respectively. Note that the payoff matrix must be expressed in terms of the rate at which cost accumulates, since the decision is continuous. Therefore, the cumulative expected uncertainty cost  $R^{\Delta t}$  represents the expected cost due to exposure to a conflict over a single measurement interval. The \$50/minute penalty (i.e.,  $\kappa_{12} - \kappa_{11}$ ) for the controller failing to intervene when a conflict exists represents the added cost to the aircraft for delaying the necessary avoidance maneuver (e.g., increased fuel-burn and flight-time).

In a continuous decision task, the expected uncertainty cost  $R(\tau)$ , defined in previous chapters, is the instantaneous rate at which cost accumulates because there is uncertainty in the state model.  $R(\tau)$  is a function of the time  $\tau$  since the

last measurement, but is independent of the absolute time, due to the assumption that the context (i.e., the payoff matrix) is constant with respect to time. The cumulative expected uncertainty cost  $R^{\Delta t}$  equals the integral of  $R(\tau)$  from  $\tau=0$  to  $\tau=\Delta t$ .

$$R^{\Delta t} = \int_0^{\Delta t} R(\tau) d\tau \quad (5.4)$$

Define  $R^{dt}$  to be the average of  $R^{\Delta t}$  over the measurement interval.

$$R^{dt}(\Delta t) = \frac{R^{\Delta t}(\Delta t)}{\Delta t} \quad (5.5)$$

Given that the decision is to continue monitoring, the instantaneous expected uncertainty cost at time  $\tau$  equals the probability that the aircraft separation  $d(\tau)$  is less than the required separation  $\theta$  times the penalty,  $\kappa_{12} - \kappa_{11} = \$50/\text{minute}$ , for the decision being incorrect.

$$R(\tau) = P[d(\tau) < \theta] (\kappa_{12} - \kappa_{11}) \quad (5.6)$$

Assume the cross-track position of each aircraft is modeled by the output of a twice-integrated, first-order Markov process. This random process is introduced in Appendix A. This model exhibits exponentially growing uncertainty as information age increases, which is appropriate when little is known about the aircraft's future trajectory. The parameter  $\sigma$  determines the rate at which the uncertainty in the state grows. Although a variety of other models could be suggested and studied at length, this model serves the objective of this example – to study the general properties of the relationship between information cost and update rate.

At a point in time, each aircraft's cross-track position is a normally distributed random variable with statistics generated by a Kalman filter, as discussed in Appendix A. Given the models for the cross-track position of each aircraft are normally distributed, a model for the aircraft separation is easily found, as shown in Appendix A. The probability of a conflict can then easily be calculated from the separation model.

Using this model, the cumulative expected uncertainty cost  $R^{\Delta t}$  increases exponentially with the length of the measurement interval. Assume the aircraft are nominally separated by 50 nautical miles. For small  $\Delta t$  (i.e., frequent updates), the cumulative expected uncertainty cost is zero, since there is zero probability a conflict will occur before the next measurement, if the aircraft were nominally separated at the time the previous measurement was taken.

#### 5.3.4 Optimal Measurement Interval

In the continuous-decision case, a new measurement has value because it affects the future cumulative expected uncertainty cost, not because it affects the expected uncertainty cost at a particular point in time, as in the discrete-decision case. Therefore, when measurements are taken periodically, the “value” of updating at a certain rate is a more meaningful metric of information management performance than the value of a single measurement. Since value must be defined relative to a baseline, the net value of updating at a certain measurement interval, relative to another measurement interval, equals the change in the total cost (i.e., the reduction in the cumulative expected uncertainty cost minus the increase in the cost of information). Notice that the net value may be negative. To compare the

**Table 5.3. Terminology used in parallel tracks example.**

Symbol	Definition
$\Delta t$	Measurement interval.
$C_T^{dt}$	Average total cost per unit of time.
$C_I^{dt}$	Average information cost per unit of time.
$R^{dt}$	Average expected uncertainty cost per unit of time.
$\kappa_{12}$	Cost per unit of time for a conflict existing and no avoidance maneuver being initiated.
$\sigma$	Parameter in the state dynamics model which determines the rate at which uncertainty in the positions of the aircraft grows.

performance of different measurement intervals, the total cost for taking measurements at a certain rate will be studied. However, to compare different measurement intervals, costs must be compared over an equal period of time. Since, for any given measurement interval, every interval is the same, due to the problem definition, the incremental (i.e., average) forms of the costs will be used. Table 5.3 explains the notation used in this example.

$$C_T^{dt}(\Delta t) = R^{dt}(\Delta t) + C_I^{dt}(\Delta t) \quad (5.7)$$

The total cost  $C_T^{dt}$  is made up of both the cost of the information  $C_I^{dt}$  and the cumulative expected uncertainty cost  $R^{dt}$ . The later is the opportunity cost, which could have been avoided if there were no uncertainty in the state model, incurred because the controller did not intervene when a conflict existed, requiring a more expensive avoidance maneuver at a later time. The units of  $C_T^{dt}$  are \$/minute.

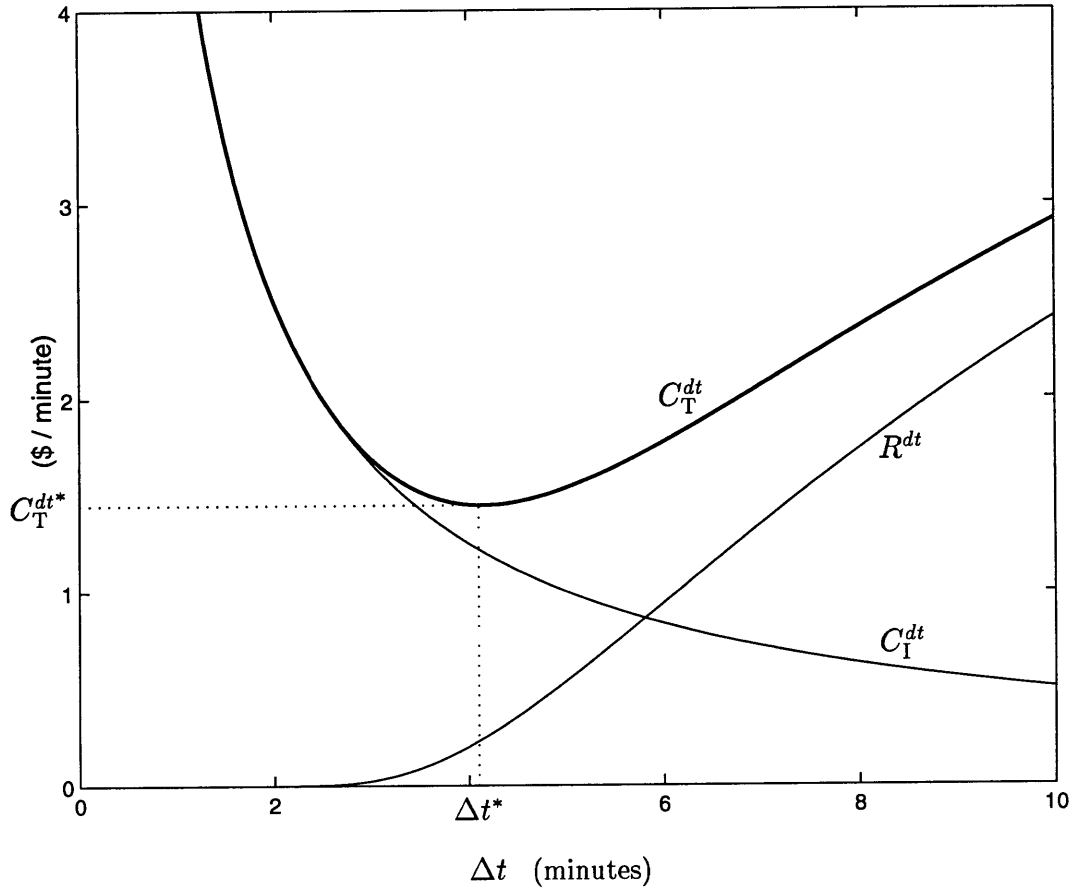
Figure 5.11 plots the per-unit-time costs which appear in Equation (5.7), as functions of the measurement interval, for the case  $C_I^{\Delta t} = \$5$ ,  $\kappa_{12} = \$50/\text{minute}$ , and  $\sigma = 10$  nautical miles/minute<sup>2</sup>. When  $\Delta t$  is small, the fixed cost for a measurement is assessed over a short time, resulting in a high cost per unit of time for information. However, the frequent measurements achieve a small average cumulative expected uncertainty cost. When  $\Delta t$  is large, the cost of information per unit of time is small, while the average cumulative expected uncertainty cost is high. Let  $\Delta t^*$  represent the optimal (i.e., minimum total cost) measurement interval, a tradeoff between the information and cumulative expected uncertainty costs.

For the assumptions made in Figure 5.11, the optimal measurement interval is approximately 4 minutes. This optimal rate balances the cost of information against the expected cost of uncertainty in the model of the aircrafts' positions. The corresponding minimum total expected cost for monitoring and maintaining separation between aircraft which are following parallel tracks separated by 50 nautical miles is approximately \$1.50/minute.

Parametric studies may be used to identify to which of the assumptions this result is most sensitive. Effort may then be concentrated on accurately identifying those parameters. The sensitivity of this result to the assumptions is studied next.

By plotting the total cost  $C_T^{dt}$  versus the measurement interval  $\Delta t$  for four values of the per-measurement cost of information  $C_I^{\Delta t}$ , Figure 5.12 illustrates the sensitivity of the optimal measurement interval to the cost of information. Increasing  $C_I^{\Delta t}$  makes information more expensive relative to the cumulative expected uncertainty cost, for any value of  $\Delta t$ . Therefore, both the optimal measurement interval  $\Delta t^*$  and the minimum total cost  $C_T^{dt*}$  increase (i.e., measurements

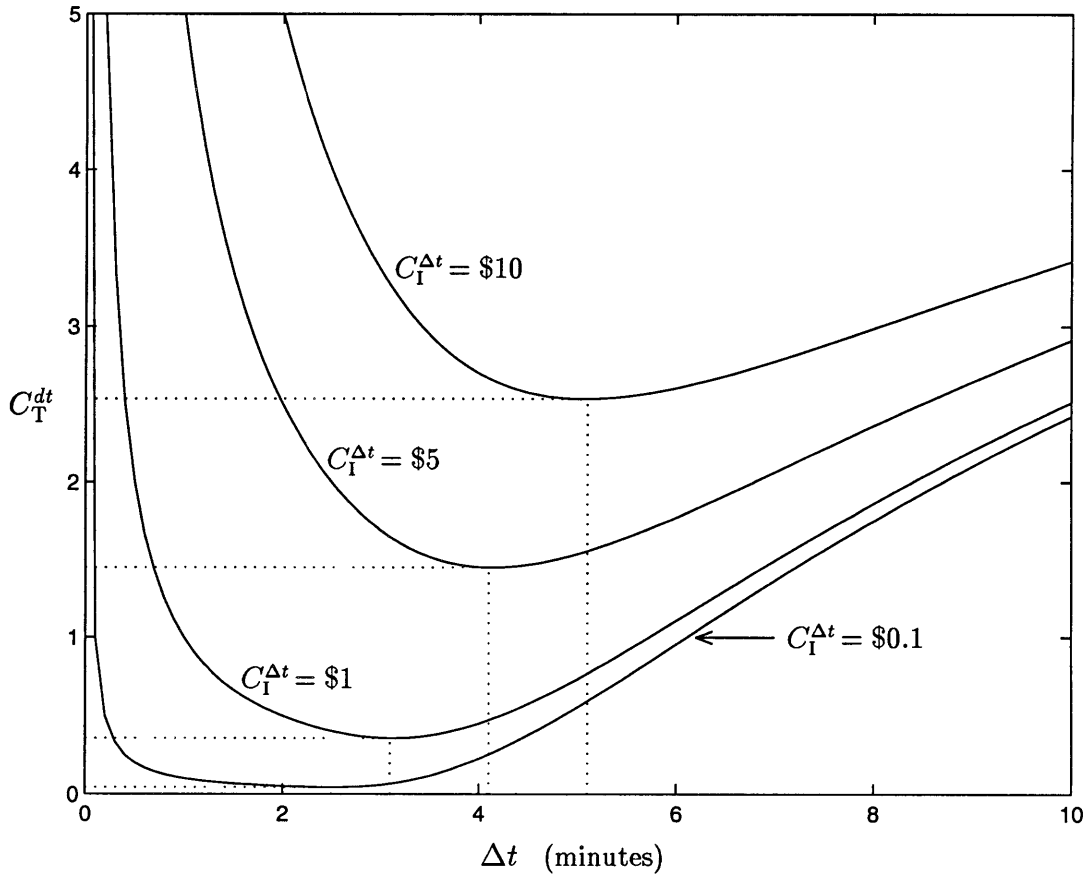




**Figure 5.11. The optimal measurement interval trades off between information and expected uncertainty costs.  $\kappa_{12} = \$50/\text{minute}$ .  $\sigma = 10$  nautical miles/minute<sup>2</sup>.  $C_I^{\Delta t} = \$5$ .**

should be taken less frequently) as  $C_I^{\Delta t}$  increases.

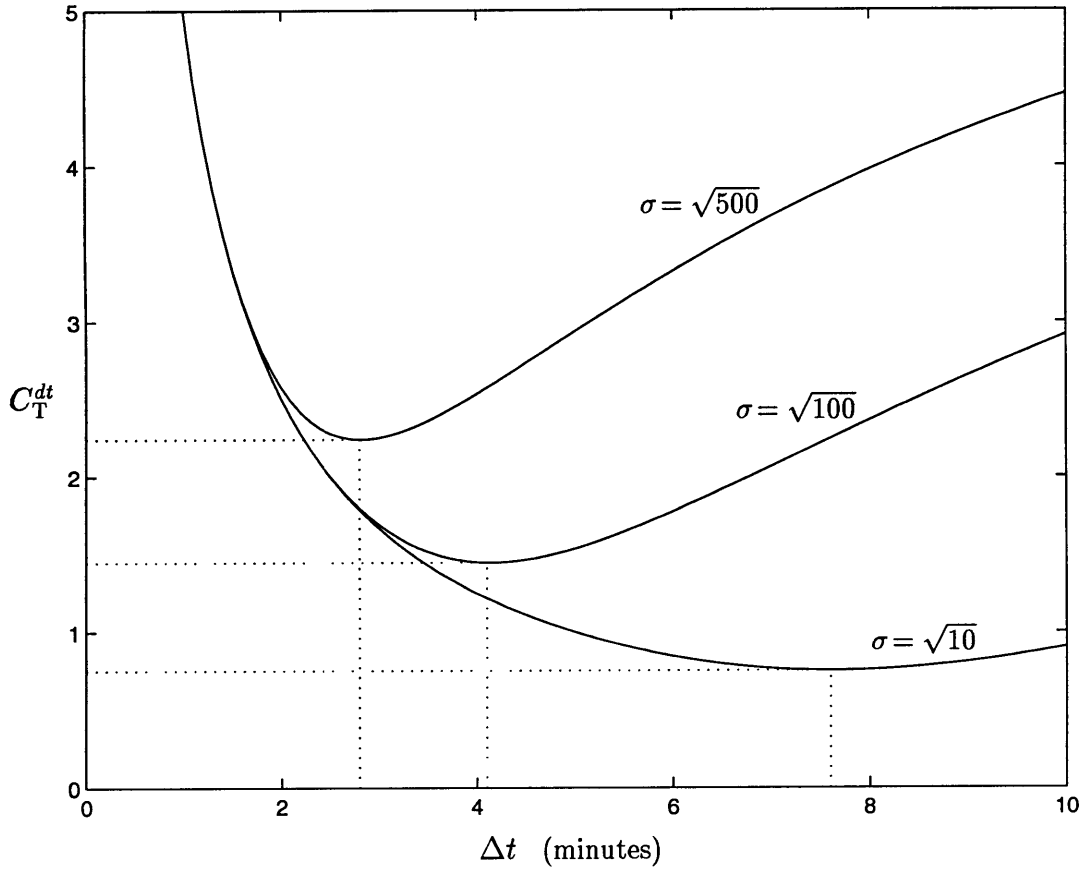
By plotting the total cost  $C_T^{\Delta t}$  versus the measurement interval  $\Delta t$  for three values of the parameter  $\sigma$ , Figure 5.13 shows the sensitivity of the optimal measurement interval to the rate at which the cumulative expected uncertainty cost grows. One of the parameters in the first-order Markov process, the mean value  $\sigma$ , determines the rate at which the variance in the state model increases with time. This parameter is used to vary the dependence of  $R^{\Delta t}$  on the length of the mea-



**Figure 5.12. Sensitivity of the optimal measurement interval to the per-measurement cost of information.** The units of  $C_T^{dt}$  are \$/minute.  $\kappa_{12} = \$50/\text{minute}$ .  $\sigma = 10$  nautical miles/minute<sup>2</sup>.

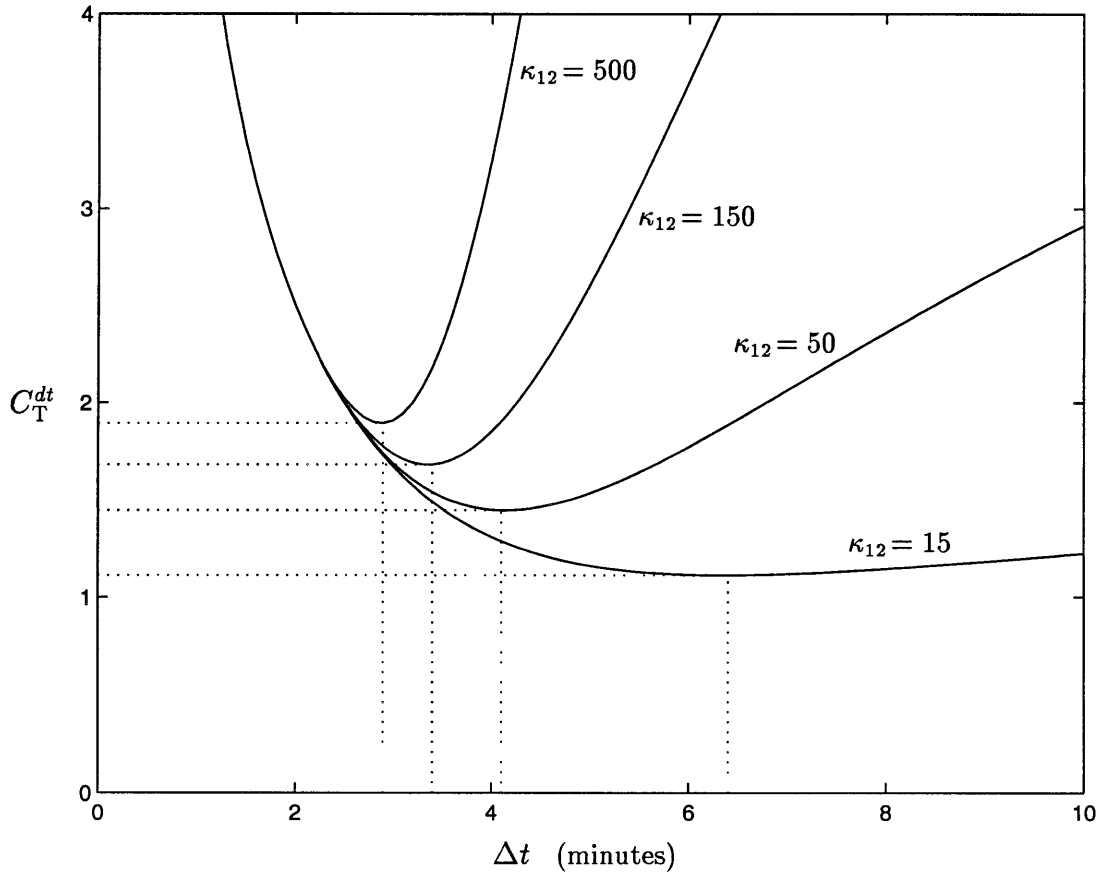
surement interval. The units of  $\sigma$  are nautical miles/minute<sup>2</sup>. As  $\sigma$  is increased, uncertainty in the cross-track aircraft positions increases faster and, consequently, the probability of a conflict increases faster. Increasing  $\sigma$  decreases the optimal measurement interval, because information is less expensive relative to the cumulative expected uncertainty cost, making seeking additional information (i.e., smaller  $\Delta t$ ) to reduce  $R^{\Delta t}$  cost effective.

By plotting the total cost  $C_T^{dt}$  versus the measurement interval  $\Delta t$  for four



**Figure 5.13. Sensitivity of the optimal measurement interval to the dependence of the cumulative expected uncertainty cost on the measurement interval.** The units of  $\sigma$  are nautical miles/minute<sup>2</sup>.  $\kappa_{12} = \$50/\text{minute}$ .  $C_1^{\Delta t} = \$5$ .

values of  $\kappa_{12}$ , Figure 5.14 shows the sensitivity of the optimal measurement interval to the penalty for continuing to monitor when an avoidance maneuver is required. Since  $\kappa_{11} = 0$ , the penalty  $\kappa_{12} - \kappa_{11}$  equals  $\kappa_{12}$ , the units of which are  $\$/\text{minute}$ . The plot is drawn for the case  $C_1^{\Delta t} = \$50$  and  $\sigma = 10$  nautical miles/minute<sup>2</sup>. As the penalty is increased, the cumulative expected uncertainty cost  $R^{\Delta t}$  increases faster with respect to the measurement interval. Consequently, the optimal measurement interval is decreased because the relative cost of information is smaller.



**Figure 5.14. Sensitivity of the optimal measurement interval to the penalty for not the incorrect decision.** The units of  $\kappa_{12}$  are \$/minute.  $C_I^{\Delta t} = \$5$ .  $\sigma = 10$  nautical miles/minute<sup>2</sup>.

These results demonstrate that the update interval should be varied depending on the cost of information relative to the cumulative expected uncertainty cost. For example, when the nominal aircraft separation is large, the cumulative expected uncertainty cost increases more slowly, so that, for small values of the measurement interval, the cost of information is large relative to the cost of not getting information. Consequently, the update rate should be low. When the aircraft separation is small, the cumulative expected uncertainty cost increases quickly. Therefore, the

total cost is minimized by updating more frequently. Note, that this situation is inherently more costly than when the aircraft separation is larger. In oceanic airspace, where satellite-based communication is expensive, the need for a high update rate when aircraft separation is small constrains the minimum separation requirement, and motivates distributing separation responsibility to the individual aircraft to avoid the high cost of using satellite-based datalinks.

### 5.3.5 Summary

The time-dependent information value model (Chapter 3) was used to identify the optimal periodic rate at which two aircraft flying along parallel tracks should broadcast position measurements. This optimal rate trades off between the expected cost of the uncertainty in the model of the aircrafts' positions and the cost of information.

The optimal measurement interval is approximately 4 minutes – when the aircraft are nominally separated by 50 nautical miles, the cost for the separation between the aircraft being less than 10 nautical miles is \$10/minute, the cost of information is \$5/measurement, and the likelihood the aircraft will deviate from their tracks is described by the given model.

Parametric studies may be used to identify to which of these assumptions the results are most sensitive. Effort may then be concentrated on accurately identifying those parameters.

The expected total cost for monitoring and maintaining separation between the aircraft is approximately \$1.50/minute. This method may be used as an analysis/design tool for studying the relationship between the total cost for monitoring

and maintaining aircraft separation and the nominal separation between oceanic tracks. If the benefit for reducing the separation between oceanic tracks can be quantified, the optimal separation can be identified.

## **5.4 Intent and Derivative Information**

The time-dependence of information value has been shown to depend on the ability of the decision-making agent to predict the future state, given aging information. The predictability of the future state depends on the nature of the random process underlying the state, which is defined by an ensemble of possible trajectories and the probabilities of each occurring. The observed state trajectory is one sample realization of the random process. How quickly the possible trajectories diverge from one another constrains how well the future state can be predicted.

The decision-making agent's ability to predict the future state depends on his model for the random process – a set of trajectories which he considers feasible and his perception of the likelihood of each occurring. A model for the random process is referred to as a model of the state dynamics. This section discusses the roles of intent and derivative information in modeling the state dynamics.

### **5.4.1 Forecasting the Trajectory of an Aircraft**

Predicting whether or not a conflict will occur between two aircraft requires a forecast for each aircraft's trajectory (more generally, a model for the dynamics of the relevant state variables), to construct models of the positions of the aircraft

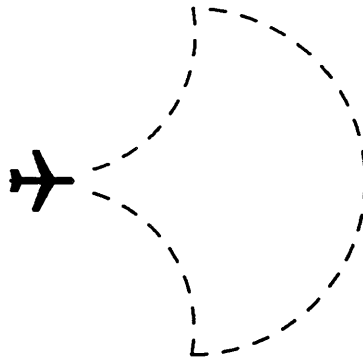
at future points in time that are of interest. This forecast depends on the available knowledge about what trajectories may occur and with what probabilities.

If no knowledge about an aircraft's future trajectory is available, the set of states which must be considered reachable by the aircraft grows rapidly as the time since the last position measurement increases, illustrated in two dimensions in Figure 5.15a. When the uncertainty in the aircraft's future position increases rapidly, conflicts cannot be predicted with both high confidence and long warning time. Moreover, because frequent aircraft position measurements will not improve the ability to confidently predict conflicts a long time before they will occur, a large initial aircraft separation is necessary to confidently predict that a conflict will not occur within a given period of time. Alternatively, if the aircraft will not deviate by more than an allowed tolerance from a known trajectory, then the aircraft's position at any future time can be predicted without any uncertainty, illustrated in Figure 5.15b, and the conflict detection problem can be solved deterministically.

An aircraft's intent can be used to forecast the aircraft's trajectory. Historically, an aircraft's intent could be inferred from its ATC clearance and the associated operating procedures. However, as free-flight initiatives increase operational flexibility, the ability to infer aircraft intent from clearances, airways, and waypoints may be lost. The role of explicitly communicating aircraft intent is to preserve the ability to forecast the aircraft's future position in a free-flight (i.e., operationally flexible) environment.

The possibility that the aircraft will fail to conform with the intent, either due to a blunder such as the aircraft failing to level off at an assigned altitude or because the aircraft's intent has changed, is captured in the forecast by assigning a level of

a.



b.



**Figure 5.15. Dependence of the set of states considered reachable on knowledge about the aircraft's trajectory.**  
Part *a*: no knowledge about the aircraft's future trajectory is available. Part *b*: the aircraft is assumed to follow a known trajectory, within some tolerance.

confidence to the intent. Whether the intent is inferred or explicitly communicated will influence this confidence. Note that the confidence in the intent does not necessarily equal the probability that the aircraft will conform to the intent, since the later is determined by the actual random process and the former is part of the model of that process.

If the forecast for the aircraft's trajectory is incorrect, then the aircraft's position at a point in time may lie outside the set of positions considered feasible at that point in time by the forecast. Such an error in the model of the aircraft's position can occur either because the model of the state dynamics neglected to include a trajectory which was permitted by the random process underlying the



state, or because the random process has changed.

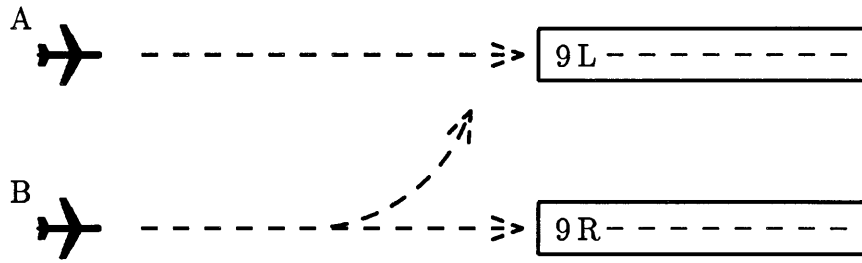
Receiving periodic measurements of the aircraft's position and its derivatives is one mechanism for detecting that the model of the state dynamics is incorrect. A second mechanism is for the aircraft to broadcast when it changes its intent – the new intent information defines the new system dynamics. The following two sections reinforce these concepts through short examples.

#### **5.4.2 Monitoring Intent Conformance using Derivative Information**

Consider two aircraft simultaneously approaching closely-spaced parallel runways. Without explicit communication, the intents of the aircraft may be inferred – to land on their assigned runways. What confidence should be placed in the aircraft conforming to this inferred intent? Note that a traffic-management decision-aid may place a higher confidence in intent information than a collision-avoidance alerting system. An aircraft's conformance with its intent must be monitored.

Periodic measurements allow the decision-making agent responsible for monitoring aircraft separation, either a controller or automaton, to detect blunders. Figure 5.16 illustrates the possibility that aircraft B may blunder toward aircraft A. More generally, the decision-making agent uses periodic updates to monitor whether the state conforms with his forecast for the state. The goal of periodically measuring derivatives of the relevant state variables is to enable the decision-making agent to detect, earlier than would be possible with measurements of only the state itself, when the state deviates from his forecast.

By providing lead information, measurements of derivatives allow a more accurate short-term prediction of the state. Consequently, these derivatives may be



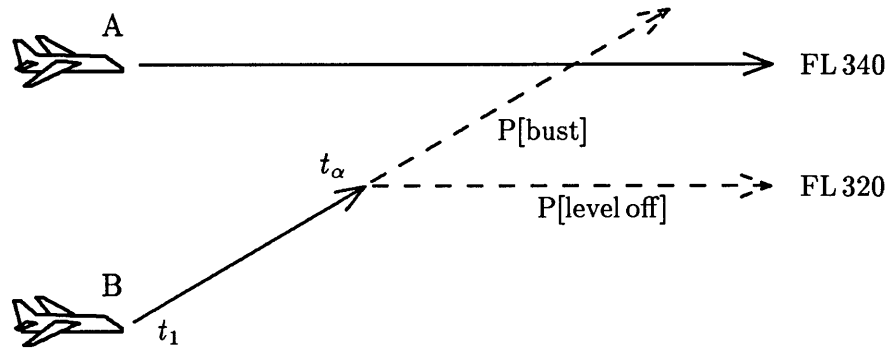
**Figure 5.16. Simultaneous approach to closely-spaced parallel runways.**

useful indicators of when the state is beginning to deviate from the forecast. In the example of monitoring the separation between aircraft simultaneously approaching closely-spaced parallel runways, measurements of aircraft heading, roll angle, or lateral acceleration, for example, may allow the controller to detect that one of the aircraft is beginning to blunder toward the other aircraft earlier than he could with only position measurement (Kuchar & Carpenter, 1998). This earlier alerting could increase the time available for the controller and pilots to respond and avoid a collision.

An advantage of using datalink to report aircraft state information is the ability to provide aircraft-measured derivative information in addition to position measurements. Although derivatives of an aircraft's position may be estimated from radar measurements of position, using knowledge of the aircraft's dynamics, the estimation process introduces a delay and amplifies noise in the measurements.

### 5.4.3 Confidence in Intent Information

Consider an aircraft B, climbing toward its assigned altitude at Flight Level (FL) 320, as shown in Figure 5.17. The aircraft's intent can be inferred from the clearance – to level off at FL 320. However, failing to level off at an assigned altitude is a common mistake.



**Figure 5.17. Uncertainty about whether or not an aircraft will level off at its assigned altitude.** Aircraft B will either level off at Flight Level 320, with probability  $P[\text{level off}]$ , or “bust” its assigned altitude, possibly conflicting with aircraft A, with probability  $P[\text{bust}] = 1 - P[\text{level off}]$ .

At time  $t_1$ , when aircraft B is well below FL 320, two models for the aircraft's future trajectory are considered feasible: the aircraft will either level off at FL 320, with probability  $P[\text{level off}]$ , or “bust” its assigned altitude, possibly conflicting with aircraft A, with probability  $P[\text{bust}] = 1 - P[\text{level off}]$ . The probability  $P[\text{level off}]$  that aircraft B levels off at FL 320 is the confidence in the intent.

The model for the altitude of aircraft B at time  $t$ ,  $f_{x_B(t)}$ , is constructed from the conditional models for the two possible trajectories,  $f_{x_B(t)|\text{level off}}$  and  $f_{x_B(t)|\text{bust}}$ ,

and the probabilities for the conditioning events,  $P[\text{level off}]$  and  $P[\text{bust}]$ .

$$f_{x_B(t)} = P[\text{level off}] f_{x_B(t)|\text{level off}} + P[\text{bust}] f_{x_B(t)|\text{bust}} \quad (5.8)$$

If aircraft B failing to level off at FL 320 may lead to a conflict with aircraft A, aircraft B's conformance with its clearance must be monitored. As the aircraft approaches the assigned altitude, what are the values of various pieces of information (e.g., altitude or vertical speed measurements, Mode Control Panel settings, or internal states of the Flight Management Computer) to a controller responsible for preventing a conflict with aircraft A?

Given new information, Bayes' theorem can be used to update the probability  $P[\text{level off}]$ , requiring the probabilities that the information would have the content that is received if each of the two models for the state dynamics were correct. Let  $t_\alpha$  be the time at which aircraft B is expected to reach FL 320. If a new measurement of aircraft B's altitude is taken prior to time  $t_\alpha$ , the measurement that is received is equally likely to result from either of the models of the state dynamics. Consequently, the measurement will not change the probability  $P[\text{level off}]$  and, therefore, has zero value.

In the process of leveling off at FL 320, aircraft B may first overshoot that altitude. Furthermore, the time at which aircraft B will reach FL 320 may not be known exactly. Therefore, in order for a new measurement taken after time  $t_\alpha$  to reveal whether or not the aircraft has leveled off, it must be delayed for some time after  $t_\alpha$ .

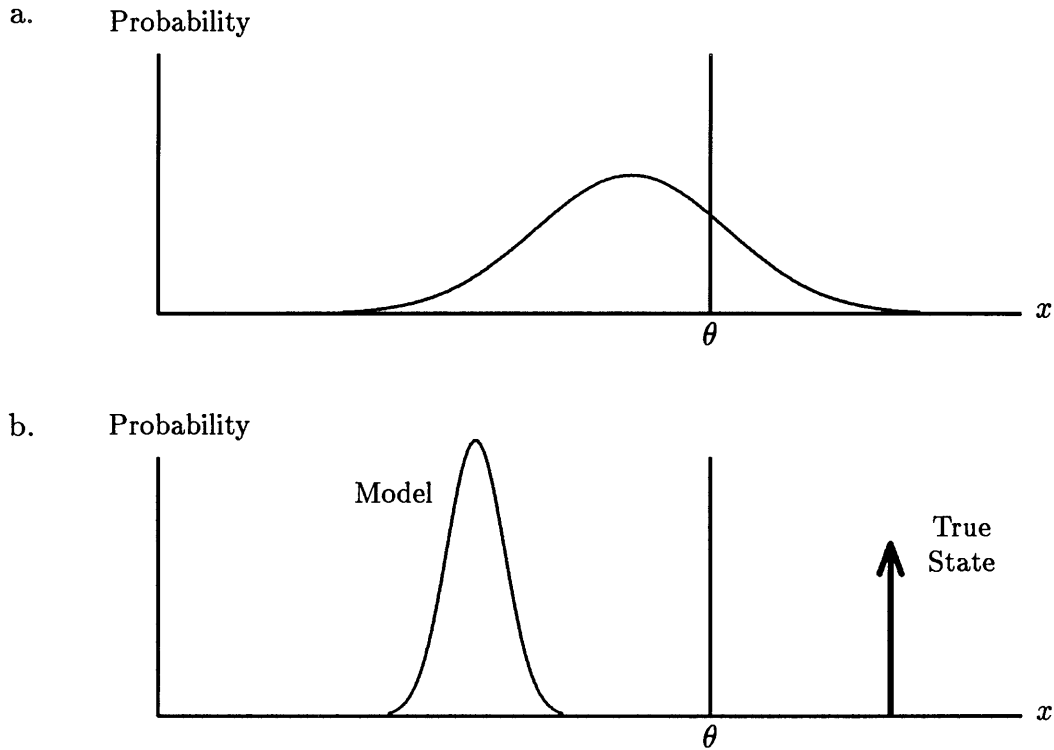
However, if aircraft B has a large vertical speed, the time between when the new measurement is taken and when the conflict would occur may not be sufficient

for the controller and pilots to react to avoid the conflict. Also, a late avoidance maneuver would be more costly than one initiated earlier. Measurements of derivatives (e.g., vertical speed) taken prior to time  $t_\alpha$  or explicit intent (e.g., Mode Control Panel settings or other internal states of the aircraft's autopilot) may allow whether or not the aircraft will level off at FL 320 to be predicted, with a high level of confidence, before time  $t_\alpha$ , increasing the available reaction time and reducing the cost of avoiding a conflict.

## 5.5 Additional Issues

### 5.5.1 Cooperative versus Non-cooperative Dependent Surveillance

This section re-visits the paradigm of cooperative versus non-cooperative information management architectures in the context of dependent aircraft surveillance (i.e., aircraft reporting information to Air Traffic Control or other aircraft via datalink). Recall that a proceduralized decision depends on a set of state variables. The decision-making agent has a model for the state variables at the current time as well as a model for how the state variables will change with time. Information that would improve the model of the state has positive value if either the error or the expected error in the model is significant in the context of the decision problem. Figure 5.18a illustrates a model of a state variable which has an expected error that is significant in the context of the decision (i.e., there is uncertainty whether the state is greater or less than the threshold  $\theta$ ). Figure 5.18b illustrates a model of a state variable which contains an error that is significant in the context of the decision.



**Figure 5.18. Expected error and error in the model of a state variable.** New information may have value when either (Part *a*) the expected error is significant in the context of the decision problem (i.e., relative to the threshold  $\theta$ ) or (Part *b*) the error is significant.

Prior to receiving new information, the decision-making agent does not know the content of the information and, therefore, can only calculate the expected value of receiving it. This expectation requires that the decision-making agent have a model of what the information content might be. Although the information-measuring agent knows the content of the new information, it does not know the decision-making agent's model of the state variable and, therefore, can only estimate the value which the information would have to the decision-making agent, using a model of the decision-making agent's model of the state variable.

The different perspectives from which the two agents value information are complimentary, allowing each to evaluate a different one of the two mechanisms shown in Figure 5.18 – the expected error in the decision-making agent’s model of the state being significant in the context of the decision problem, or the error being significant – to determine whether new information would have value. The information-measuring agent broadcasts updates aperiodically when it expects the error in the decision-making agent’s state model is significant in the context of the decision problem. The information broadcast by the information-measuring agent allows the decision-making agent to detect when it’s model of the state dynamics is incorrect. In a cooperative dependent surveillance architecture, the decision-making agent requests periodic updates to maintain a small expected error (i.e., uncertainty) in its model for the state. This rate depends on the ability to forecast the future state (i.e., the model of the state dynamics). In a non-cooperative dependent surveillance architecture, because the decision-making agent’s model for the state dynamics may be incorrect (i.e., the state may do something the decision-making agent does not expect), the decision-making agent must also use periodic updates to detect unexpected events (i.e., model error in its model of the state dynamics). The necessary update rate is determined by the acceptable latency in detecting unexpected events.

Assume the information-measuring agent is an aircraft, which is reporting its current state and, possibly, intent to an air traffic controller, and the decision-making agent is the controller, who is responsible for maintaining safe separation between this and other proximate aircraft. The decision problem is whether or not intervention is necessary to prevent a conflict (i.e., a violation of the minimum

separation requirement). The controller has a model of the aircraft's position at the current time as well as a model of how the position will change in the future (i.e., the state dynamics). The controller's model of the aircraft dynamics may be based on received intent information or an assumption, such as the aircraft will fly straight and level at a constant velocity. In a non-cooperative dependent surveillance environment, the controller must request all of the information which he receives from the aircraft. Therefore, even if the controller has received information about the aircraft's intent at a previous time, the controller must continually request new information to detect whether or not the aircraft's intent has changed. Therefore, periodic updates are required to maintain a small (measured in the context of the decision problem) expected error in the controller's model of the aircraft's position.

In a cooperative dependent surveillance environment, the aircraft can, when it believes a piece of information may have sufficient value to the controller, report this information unsolicited. If the aircraft does not know the controller's model for its position, it can only estimate when the error between the controller's model and the new information (i.e., a current measurement of its positions) is significant. However, if the aircraft has told the controller its intent, then it knows the controller's model for how its position will change in time. Therefore, the aircraft can know when new information may (depending on whether or not there are other aircraft in the vicinity) have positive value to the controller – when the aircraft deviates from or changes its intent.

If the controller trusts that the aircraft will always report changes in its intent, then the controller's uncertainty is limited to the navigation and guidance performance of the aircraft, which is small compared to the required separation, and the

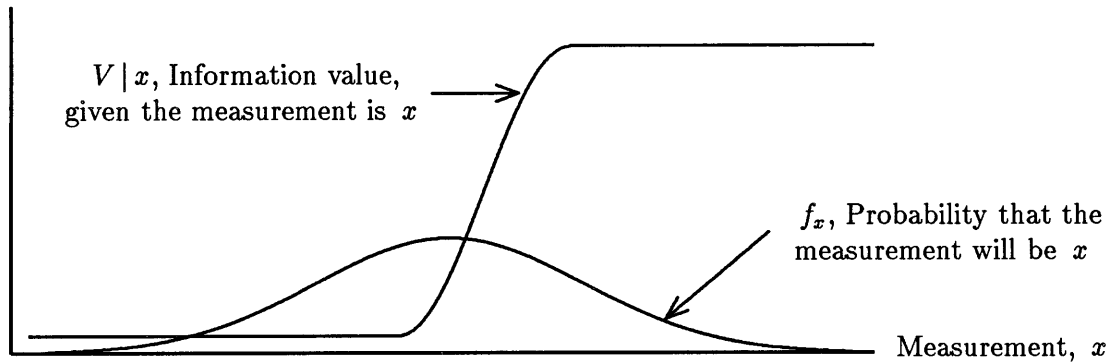


controller can predict future conflicts using the reported intent as a deterministic model for the aircraft's future trajectory. In this way, a cooperative dependent surveillance architecture can provide efficient detection of errors in the controller's model of the aircraft's position, without periodic updates.

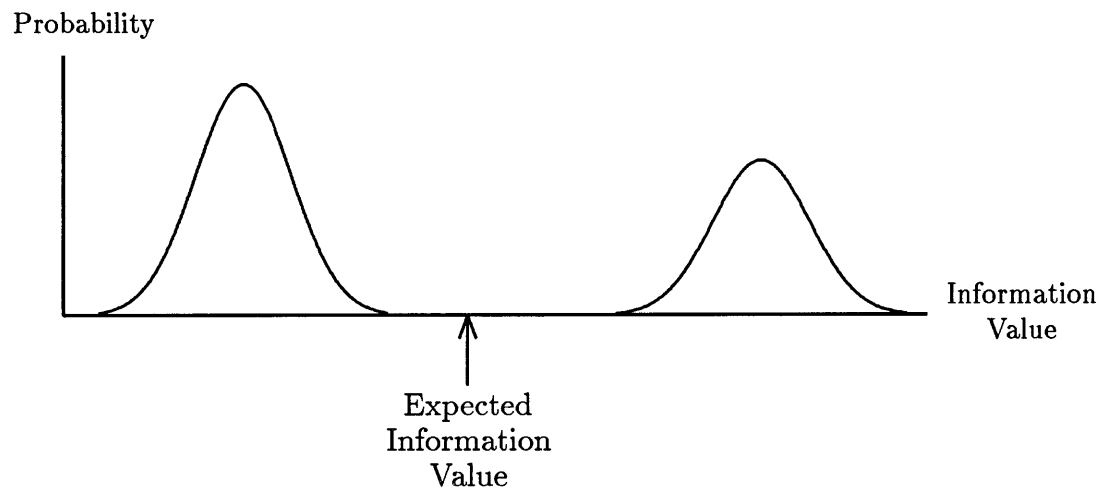
However, in a safety critical application (e.g., a collision avoidance alerting system rather than a traffic management decision-aid), the aircraft's conformance with its reported intent must be monitored, requiring that the controller receive periodic measurements of the aircraft's state. If the cost of providing these frequent measurements to the controller via datalink is large, either due to a direct cost for satellite service in oceanic airspace or an indirect cost resulting from the available bandwidth being limited in domestic airspace, then responsibility for collision avoidance could be shifted to the aircraft. The Traffic Alert and Collision Avoidance System (TCAS) is an example in which this has been done. Datalink communication directly between two proximate aircraft can be less expensive because less infrastructure (e.g., a satellite) is required, and the bandwidth is shared between only a few aircraft in a small spatial area and may be re-used elsewhere.

### 5.5.2 Uncertainty in the Value of Information

The expected information value metric does not indicate the variability that can occur in the information value of a measurement. This section discusses uncertainty in information value and its implication for information management. Consider a situation in which a measurement may take on a wide range of values, illustrated by the broad probability density function  $f_x$  in Figure 5.19. Assume that the nature of the decision problem and the a priori model of the state are such



**Figure 5.19.** Conditional information value and probability density for a hypothetical measurement.



**Figure 5.20.** Probability density function for the information value of the hypothetical measurement.

that some of these possible values of the measurement would have high information value if they occurred, and others would have low information value. Figure 5.19 illustrates a possible plot of conditional information value  $V|x$  as a function of the feasible values of the measurement for such a situation.

Figure 5.20 illustrates the corresponding probability density function for the

information value of the measurement. The information value of the measurement will be either larger or smaller than the expected information value. This variability in the information value of the measurement may be significant to the decision of whether or not to seek the measurement. A class of such applications will be discussed in the following section.

### **5.5.3 Alerting Systems versus Decision-Aids**

When the elements of the payoff matrix are homogeneous (i.e., of roughly the same order of magnitude), which is typical of decision-aiding applications, the value of information can be neither much greater than nor much less than the expected value. In this case, expected information value may be a useful metric for making decisions concerning seeking or using information. However, when the payoff matrix contains disproportionate elements, which is typical when information is applied to risk analysis or in an alerting system, there can be a very small probability that the information will have a very large value, while the expected information value remains small. In this case, expected information value may not be a sufficient basis on which to manage information, and the probability density function for conditional information value should be considered.

To illustrate this point, consider the problem of calculating the value to a collision avoidance system of measurements of an aircraft's position. The cost of the collision avoidance system intervening when no collision would have occurred is relatively small – a small increase in fuel and flight-time. In contrast, the cost of the collision avoidance system failing to intervene when a collision will occur is extremely large – two aircraft collide. Table 5.4 shows a generic payoff matrix for

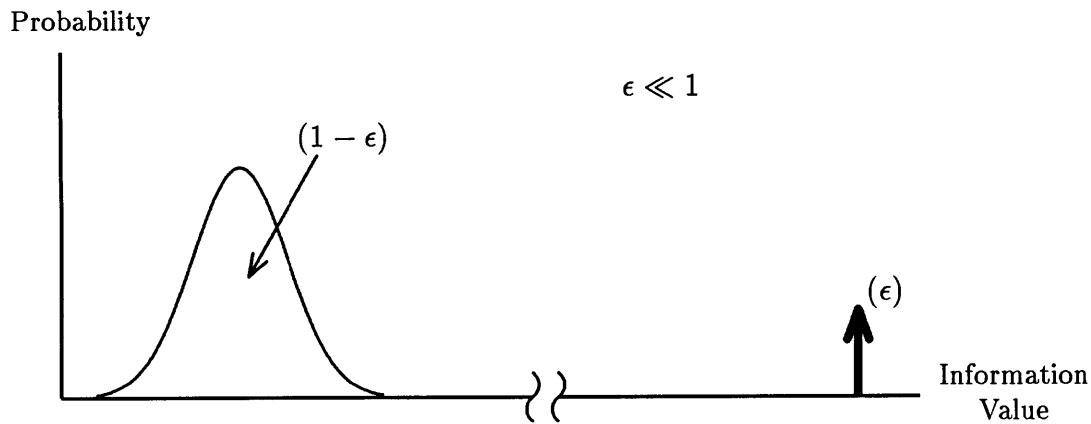
an alerting system, in which the penalties for the two possible wrong decisions are disproportionate.

**Table 5.4. General structure of the payoff matrix when the application of information is in an alerting system. The penalties for the two wrong decisions are disproportionate.**

Event	Collision Avoidance System	
	No Alert	Alert
No Collision	0	Small
Collision	Extremely Large	Small

Assume the collision avoidance system knows the aircraft's intent, which there is a high probability the aircraft will follow. Measurements of the aircraft's current position, that confirm the aircraft is following the forecast trajectory, have low values because the measurements, which are expected, do not significantly change the collision avoidance system's model of the aircraft's position. In contrast, a measurement which would allow the collision avoidance system to detect that the aircraft is deviating from the forecast trajectory (e.g., blundering) may have a high value, if there is another aircraft nearby with which a collision might occur. However, the probability of the aircraft deviating from the forecast trajectory is very low.

Figure 5.21 illustrates the structure of the probability density function for the conditional information value in a problem of this type. There is a high probability



**Figure 5.21. Small probability of information having very large value.**

that the new measurement will have a relatively small value, but there is a non-zero probability,  $\epsilon$ , that the value of the measurement will be very large. The expected value of the measurement will be very low compared to the value of the high-value measurement, because the probability of receiving a high-value measurement is very small. The expected value will also be sensitive to the relative orders of magnitude of the elements in the payoff matrix and the probabilities, both of which are difficult to determine. Calculating the probability of an extremely rare event, such as an aircraft blundering or two aircraft colliding, is problematic due to the small sample size. Assigning a cost to an aircraft collision is speculative.

If the expected value of a new measurement is low but an infrequent, high-value measurement cannot be missed (e.g., for safety reasons), then a measure of average information value may not be a sufficient basis for making information management decisions. *Decision Analysis* addresses this issue by introducing a non-linear *utility function* to transform the probability density function for the possible *values* of a

*lottery* into a scalar index, *utility*, of the desirability of the lottery to the decision maker. For the special case in which the utility function is linear, the lottery's utility equals its expected value. A non-linear utility function can be used to make the utility of a lottery which has a small probability of having a very large value, larger, for example, than its expected value.

A non-linear utility function is one approach to making information management decisions in the presence of uncertainty about the value of the information. Many alternate approaches can be suggested. For example, a more conservative approach is to base information management decisions on the maximum possible value which information could have. Regardless of the approach taken, the probability density function for the conditional information value fully describes the possible consequences of seeking or forgoing the measurement.

#### **5.5.4 The Value of Information which Leads to an Unsuccessful Alert**

Assume that an aircraft will encounter a hazard along its nominal trajectory. Kuchar (1995) recognized that although an avoidance trajectory may miss the initial hazard, it may encounter other hazards which would not have been encountered along the initial trajectory. Assume that if an alert is issued, the aircraft will encounter a second, more costly hazard along the avoidance trajectory (i.e., the cost of encountering the hazard that is along the avoidance trajectory is higher than the cost of encountering the initial hazard). What is the value to an alerting system of new information that reveals which a hazard encounter will occur along the initial trajectory, but which does not reveal the presence of the hazard along the avoidance trajectory?

Assume that the new information results in an alert being issued, and the avoidance trajectory being followed. Knowledge of the information by the alerting system increases the expected cost, since the aircraft will encounter the more severe hazard. Therefore, an information value metric based on the expected cost resulting from the alerting decision would attribute negative value to this information. However, forgoing the information and not alerting, because the presence of a hazard along the initial trajectory is not known, is not the correct solution to the information management problem.

The current information value model attributes positive value to the new information, because knowledge of it improves the alerting system's model of the world – improving the prediction of whether or not a hazard encounter will occur along the initial trajectory. That the expected cost to the aircraft remains high after the alert is issued, due to the fact that a hazard encounter will still occur, implies that additional information should be sought to reveal the presence of the second hazard and allow a different avoidance trajectory to be chosen.

“There are other Annapurnas in the lives of men.”

Maurice Herzog – on the first ascent, with Louis Lachenal,  
of a peak over 8000 meters, 1950.



## Chapter 6

# Conclusions

### 6.1 Summary

The introduction of aeronautical datalinks as an alternate communication modality will enable direct connectivity between airborne and ground-based computers. The resulting ability of automation to accomplish communication tasks without human intervention will make feasible a variety of decision-support systems that require the communication of time-varying information. Historically, manual procedures for managing the dissemination of information by and to pilots via voice communication have been developed heuristically. This thesis proposed a generalized approach to designing human/automation mechanisms, both automatic systems and manual procedures, for efficiently managing the collection, dissemination, or presentation of time-varying information, when the sensor, communication, or display resources are constrained.

In particular, three questions were addressed. How does the age of information affect its ability to support a proceduralized decision problem? If a new measurement is taken, how does the value of that information vary with the time at which the measurement is taken? How frequently (periodic) or under what conditions (aperiodic) should information be updated to support a proceduralized decision problem? To answer these questions, a novel model of time-dependent information value, which combines elements of classic information value theory with estimation techniques, was developed for a class of “proceduralized” decision problems.

## **Proceduralized Decision Problems**

Proceduralized decision problems are characterized by the existence of an established procedure or rule which specifies the correct decision or action as a function of one or more relevant state variables. Threshold surfaces were introduced to model the way in which a proceduralized decision problem depends on a set of relevant state variables. The solution of a proceduralized decision problem is found by solving an estimation problem – identifying the region of the state space in which the state vector.

Knowledge about a relevant state variable (i.e., situation awareness) was modeled by a probability density function describing the perceived likelihood the state will take on each of the possible values. This model captures both the state estimate and the confidence/uncertainty in that estimate. A model of the random process driving the state (i.e., the state dynamics) was used to describe the ability to forecast the state at future times. A payoff matrix and a model of the decision-making

agent's decision rule were used to model the context within which information is valued.

## Information Value

For the class of proceduralized decision problems, the value which a piece of information has to the decision-making agent is defined as the change which knowledge of the information effects in the probability that the decision-making agent will choose a sub-optimal decision, weighted by the amount by which the cost of the sub-optimal decision exceeds the cost of the optimal (i.e., minimum cost) decision. The decision-making agent may choose a sub-optimal decision due to uncertainty in the model of the state variables on which the procedure depends. Information improves the decision-making agent's model of these state variables.

This definition is subtly different from the classic *expected value* definition, which measures the effect of the information directly on the cost resulting from the decision. Given that proceduralized decision problems are estimation/prediction problems, the goal of providing information is to improve the decision-making agent's ability to model the relevant state variables. The motivation for introducing a different information value metric is to directly measure, in the context of the proceduralized decision problem, the impact of information on the ability to model the relevant state variables,

The ability to model the relevant state variables was measured by the expected cost of the uncertainty in the state model – the amount by which the cost resulting from the decision will exceed the minimum achievable cost, due to the uncertainty causing the decision-making agent to choose a sub-optimal decision. In this way,

the significance of uncertainty to a proceduralized decision problem is distinguished from the magnitude of the uncertainty.

The flexibility of this information value model allows it to be applied from the perspective of the decision-making agent, an external information-valuing agent, or the designer of an information management process, depending on the component models used for each element of the information value model.

### **Case Study – Airport Surface Observations**

The measurement and dissemination of airport surface observations, to support an early decision whether or not to divert to an alternate airport, was examined. The example was used to demonstrate the formulation of a problem and the calculation of expected information value. Results provided insight into the time-dependence of information value and its implications for designing information management processes.

### **Case Study – Aircraft on Crossing Trajectories**

When aircraft should update state information in a dependent surveillance environment, to support a controller monitoring and maintaining separation, was studied in two encounter geometries.

The model of time-dependent information value (Chapter 3) was generalized to calculate the collective expected information value of multiple new measurements. Given the per-measurement cost of information, the model may be used to determine how many measurements should be taken, and at what times.

An encounter between two aircraft on intersecting trajectories was studied. The time intervals between optimally spaced measurements were shown to decrease as the time remaining until the possible conflict decreases. The cumulative information value, expected prior to receiving any of the measurements, exhibited diminishing returns with an increasing number of measurements.

After each new measurement is received, the new knowledge may change the expected value of future information. The same method may be used to re-plan how many and when future measurements should be taken. The computational demand of re-calculating when to take future measurements each time a measurement is received make the approach best suited as an off-line analysis/design tool.

### **Case Study – Aircraft on Parallel Tracks**

The time-dependent information value model (Chapter 3) was used to identify the optimal periodic rate at which two aircraft flying along parallel tracks should broadcast position measurements. This optimal rate trades off between the expected cost of the uncertainty in the model of the aircrafts' positions and the cost of information.

The optimal measurement interval is approximately 4 minutes – when the aircraft are nominally separated by 50 nautical miles, the cost for the separation between the aircraft being less than 10 nautical miles is \$10/minute, the cost of information is \$5/measurement, and the likelihood the aircraft will deviate from their tracks is described by the given model. Parametric studies may be used to identify to which of these assumptions the results are most sensitive. Effort may then be concentrated on accurately identifying those parameters.

The corresponding minimum total expected cost for monitoring and maintaining separation between aircraft which are following parallel tracks separated by 50 nautical miles is approximately \$1.50/minute. This method may be used as an analysis/design tool for studying the relationship between the total cost for monitoring and maintaining aircraft separation and the nominal separation between oceanic tracks. If the benefit for reducing the separation between oceanic tracks can be quantified, the optimal separation can be identified.

## 6.2 Conclusions

### Periodic Update Rate

Information should be updated when either:

1. The **expected error** (i.e., uncertainty) in the decision-making agent's model of the state is significant in the context of the proceduralized decision problem.

and

New information will reduce the expected error in the state model.

or

2. The **error** in the decision-making agent's model of the state is significant in the context of the proceduralized decision problem.

and

New information will reduce the error in the state model.

The decision-making agent's model for the state dynamics may be wrong. The implication of this is that, in order to be able to detect the occurrence of an unexpected event (i.e., an error in the state model due to model-error in the model

of the state dynamics), information must be updated periodically. The necessary update rate is determined by the acceptable latency in detecting unexpected events. A cooperative approach to managing information has been shown to reduce the burden of detecting unexpected events.

Minimum aircraft separation requirements are constrained by the possibility of a blunder and the latency in the blunder being detected and the non-intruder aircraft performing an avoidance maneuver. Therefore, there is a tradeoff between the information update rate and the separation requirements.

The necessary periodic update rate also depends on the rate at which the expected error (i.e., uncertainty) in the decision-making agent's state model increases, measured in the context of the proceduralized decision problem. This rate (i.e., the inverse of the information persistence) is determined by the ability to forecast the future state (i.e., the model of the state dynamics). Intent or derivative information can improve the ability to forecast the future state trajectory.

## **Derivative and Intent Information**

Derivative and intent information reduce the required periodic update rate by modifying the model of the state dynamics so as to reduce the rate at which uncertainty grows (i.e., by improving the ability to confidently forecast the state farther into the future).

In the absence of restrictive operating procedures, the role of explicitly communicating aircraft intent is to preserve the ability to forecast the aircraft's future position in a free-flight (i.e., operationally flexible) environment, with both high

confidence and long look-ahead time. The structure of the airspace (e.g., clearances and airways) has traditionally enabled this forecasting.

The role of derivative information is to improve the ability to monitor conformance of the state with the forecast (e.g., to allow earlier detection of blunders), by providing lead information.

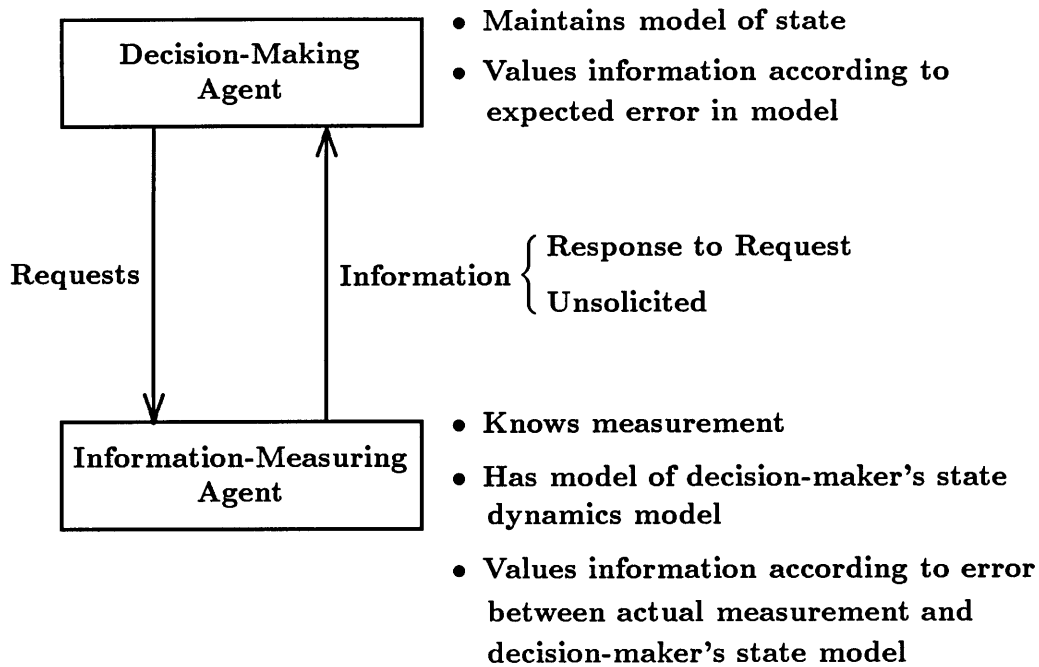
### **Cooperative versus Non-cooperative Architectures**

In a *non-cooperative* information management environment, the decision-making agent must initiate all information transfer. Since the decision-making agent does not know the content of the information prior to receiving it, in order to detect unexpected events, the decision-making agent must request updates periodically.

A *cooperative* information management environment is one in which agents at both the information collection and decision-making ends of the information transfer share responsibility for managing information. The decision-making agent requests information. The information-measuring agent, who can use knowledge of the information content in measuring the value which that information would have to the decision-making agent, announces information unsolicited.

The different perspectives from which the two agents value information are complimentary, each suited to evaluate a different one of the two mechanisms (summarized at the start of this section) by which information can have value. In a cooperative environment, the decision-making agent requests periodic updates due to the expected error (i.e., uncertainty) in its model of the state. The information-measuring agent broadcasts updates aperiodically when it expects the error in the





**Figure 6.1. Roles of Agents in Cooperative Information Management.**

decision-making agent's state model is significant in the context of the proceduralized decision problem. This paradigm for the roles of the two agents is illustrated in Figure 6.1.

In this way, a cooperative information management environment can reduce the required periodic update rate by providing efficient detection of unexpected events (i.e., errors in the decision-making agent's model of the state dynamics). However, to measure the error in the decision-making agent's state model, the information-measuring agent must know that model. If the decision-making agent does not explicitly provide its model for the state dynamics, the information-measuring agent should assume a simple model (e.g., the decision-making agent believes the state will remain constant) so that the decision-making agent is able

to predict under what conditions the information-measuring agent will provide unsolicited information, reducing the burden of detecting unexpected events through periodic requests for updates.

## **Context**

To maintain situation awareness in a dynamic environment (i.e., an environment in which information is time-dependent), information must be repeatedly updated. However, the value of information depends strongly on the context in which it is measured. Since the context typically changes with time, the periodic rate at which information is updated should be adjusted to always equal the rate appropriate for the current context. Updating information at a constant rate, irrespective of the current context, is inherently inefficient because it requires using the highest necessary update rate at all times.

## **Decision-aiding versus Alerting Systems**

Expected information value is a useful metric for making decisions concerning seeking or using information when the elements of the payoff matrix are homogeneous, which is typical of decision-aiding applications of the information. When the payoff matrix contains disproportionate elements, which is typical when information is applied to risk analysis or in an alerting system, there can be a very small probability that the information will have a very large value, while the expected information value remains small. In this case, expected information value may not be a sufficient basis on which to manage information, and the probability density function of conditional information value should be considered.

# Appendix A

## Modeling and Estimation

The information value calculations in the previous chapters assumed models for the dynamics of the state variables. Section A.1 introduces the two random processes that were used to model the state variable dynamics. Chapters 3 and 4 used an integrated, first-order Markov process, and Chapter 5 used a twice-integrated, first-order Markov process. Note that many other models for these physical processes could be proposed, each with different advantages. The information value calculations also require the ability to construct models (i.e., probability density functions) for the state variables at specific points in time, given the available information and the model of the state dynamics. Section A.2 introduces the discrete-time Kalman filter algorithm for this purpose. Section A.3 discusses the approach used in Chapter 5 to calculate the probability that a conflict will occur between two aircraft. A variety of alternative algorithms are available.

## A.1 First-order Markov Processes

The first-order Markov process is characterized by two parameters: the mean-square value  $\sigma^2$  determines the magnitude of the output in response to the unity white noise input  $w(t)$ , and the time constant  $\beta$  determines the rate at which the autocorrelation function approaches zero. The autocorrelation function  $R(\tau)$  and spectral density function  $S(s)$  for a first-order Markov process appear in Equations (A.1) and (A.2), respectively.<sup>1</sup> The exponential form of the autocorrelation function indicates that as the time separation between two outputs of the process increases, the outputs become less correlated (Brown, 1983).

$$R(\tau) = \sigma^2 e^{-\beta|\tau|} \quad (\text{A.1})$$

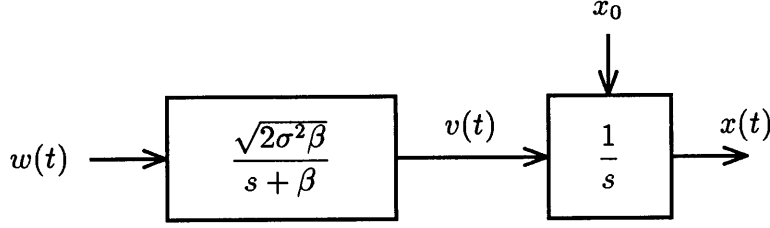
$$S(s) = \frac{2\sigma^2\beta}{-s^2 + \beta^2} = \left( \frac{\sqrt{2\sigma^2\beta}}{-s + \beta} \right) \left( \frac{\sqrt{2\sigma^2\beta}}{s + \beta} \right) \quad (\text{A.2})$$

### A.1.1 Integrated First-order Markov Process

Figure A.1 illustrates an integrated, first-order Markov process. The state dynamics are described by Equation (A.3). The output  $x(t)$  is a random process characterized by a constant expected value and exponentially increasing uncertainty.

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<sup>1</sup> To conform to the notation used elsewhere, the symbol  $R$ , which has previously been introduced as the *expected uncertainty cost*, is also used as the *autocorrelation function*, as well as the *covariance of the measurement noise* in the next section. The meaning of  $R$  should always be clear from the context.



**Figure A.1. Integrated, first-order Markov process.**  
 $w(t)$  is unity white noise.

$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\beta \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ \sqrt{2\sigma^2\beta} \end{bmatrix} w \quad (\text{A.3})$$

A discrete-time representation is derived to facilitate using a discrete-time Kalman filter. The discrete-time formulation of the Kalman filter is a more convenient form of the estimator when measurements are incorporated only occasionally. State-space methods may be used to write the solution of the differential equation (A.3) as a difference equation, relating the state at time  $t_{k+1}$ ,  $\mathbf{x}(t_{k+1})$ , to the state at time  $t_k$ ,  $\mathbf{x}(t_k)$ , (Brown, 1983).

$$\mathbf{x}(t_{k+1}) = \Phi(t_{k+1}, t_k) \mathbf{x}(t_k) + \int_{t_k}^{t_{k+1}} \Phi(t_{k+1}, \tau) G(\tau) w(\tau) d\tau \quad (\text{A.4})$$

$\Phi(t_{k+1}, t_k)$  is the state transition matrix. For stationary systems, such as the present,  $\Phi(t_{k+1}, t_k)$  depends only on the time interval  $\Delta t = t_{k+1} - t_k$  (i.e., is independent of  $t_k$ ), and may be rewritten as  $\Phi(\Delta t)$ . Gelb (1974) gives the state transition matrix for this system.

$$\Phi(\Delta t) = \begin{bmatrix} 1 & \frac{1}{\beta}(1 - e^{-\beta\Delta t}) \\ 0 & e^{-\beta\Delta t} \end{bmatrix} \quad (\text{A.5})$$

Equation (A.4) is typically rewritten in a more compact form.

$$\mathbf{x}_{k+1} = \Phi \mathbf{x}_k + \mathbf{w}_k \quad (\text{A.6})$$

$\mathbf{w}_k$  is the time-wise uncorrelated, zero-mean sequence (i.e., the autocorrelation function equals zero everywhere except at  $\tau = 0$ ) representing the contribution of the continuous process noise  $w(t)$  over the interval  $t_k$  to  $t_{k+1}$  to the state at time  $t_{k+1}$ . Let  $Q$  be the covariance of  $\mathbf{w}_k$  (i.e., the contribution of the process noise over an interval of length  $\Delta t$  to the error covariance  $P$ ). The Kalman filter algorithm computes the error covariance matrix  $P$  as a measure of the uncertainty in the prediction of the state.  $Q$  is required in this calculation.  $Q$  is time-invariant when  $w(t)$  is stationary, and is symmetric (Brown, 1983).

$$\mathbb{E}[\mathbf{w}_k \mathbf{w}_i^T] = \begin{cases} Q & \text{if } i = k \\ 0 & \text{otherwise} \end{cases} \quad (\text{A.7})$$

where

$$Q = \begin{bmatrix} \mathbb{E}[x x] & \mathbb{E}[x v] \\ \mathbb{E}[v x] & \mathbb{E}[v v] \end{bmatrix} \quad (\text{A.8})$$

and the elements of  $Q$  are given in Brown (1983).

$$\mathbb{E}[x x] = \frac{2\sigma^2}{\beta} \left[ \Delta t - \frac{2}{\beta} (1 - e^{-\beta \Delta t}) + \frac{1}{2\beta} (1 - e^{-2\beta \Delta t}) \right] \quad (\text{A.9a})$$

$$\mathbb{E}[x v] = \mathbb{E}[v x] = 2\sigma^2 \left[ \frac{1}{\beta} (1 - e^{-\beta \Delta t}) - \frac{1}{2\beta} (1 - e^{-2\beta \Delta t}) \right] \quad (\text{A.9b})$$

$$\mathbb{E}[v v] = \sigma^2 (1 - e^{-2\beta \Delta t}) \quad (\text{A.9c})$$

### A.1.2 Twice-integrated First-order Markov Process

Figure A.2 illustrates a twice-integrated, first-order Markov process. The acceleration  $a(t)$  is a random variable modeled by a first-order Markov process, Equation (A.9a). Velocity  $v(t)$  is a random variable modeled as the sum of the integral of the acceleration (a random variable with the statistics of an integrated first-order Markov process) and the initial velocity  $v_0$  (a constant), Equation (A.9b). The random variable position  $x(t)$  is found by integrating velocity and adding the initial position  $p_0$ , Equation (A.9c).

$$\dot{a}(t) = -\beta a(t) + \sqrt{2\sigma^2\beta} w(t) \quad (\text{A.10a})$$

$$\dot{v}(t) = a(t) \quad (\text{A.10b})$$

$$\dot{x}(t) = v(t) \quad (\text{A.10c})$$

These three equations may be rewritten in state-space notation.

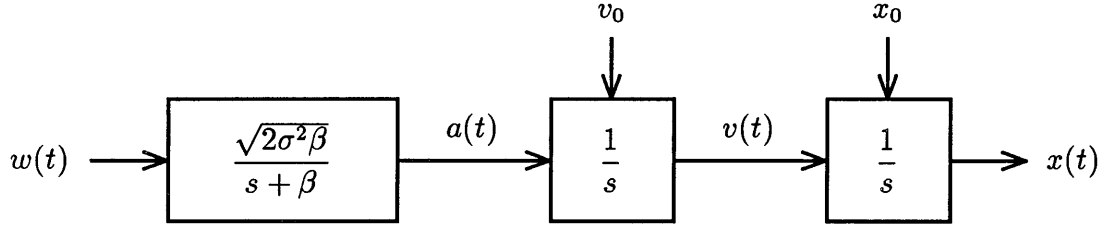
$$\dot{\mathbf{x}}(t) = F \mathbf{x}(t) + G w(t) \quad (\text{A.11a})$$

$$\mathbf{x}(t) = \begin{bmatrix} x(t) \\ v(t) \\ a(t) \end{bmatrix} \quad F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\beta \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ 0 \\ \sqrt{2\sigma^2\beta} \end{bmatrix} \quad (\text{A.11b})$$

The state transition matrix  $\Phi$  may be found by expanding (A.12a) using  $F$  from (A.11b).

$$\Phi(\Delta t) = e^{F \Delta t} \quad (\text{A.12a})$$

$$= \begin{bmatrix} 1 & \Delta t & \frac{1}{\beta^2}(\beta\Delta t - 1 + e^{-\beta\Delta t}) \\ 0 & 1 & \frac{1}{\beta}(1 - e^{-\beta\Delta t}) \\ 0 & 0 & e^{-\beta\Delta t} \end{bmatrix} \quad (\text{A.12b})$$



**Figure A.2. Twice-integrated first-order Markov process.**  
 $w(t)$  is unity white noise.

The covariance of the process noise,  $Q$ , is a three-by-three symmetric matrix.

$$Q = \begin{bmatrix} E[a a] & E[a v] & E[a x] \\ & E[v v] & E[v x] \\ & & E[x x] \end{bmatrix} \quad (\text{A.13})$$

To demonstrate the method for calculating the elements of  $Q$ ,  $E[a a]$  will be derived.  $E[a a]$  is the contribution of the process noise over an interval of length  $\Delta t$  to the uncertainty in the acceleration estimate. The transient response of a linear system to a forcing function can be written as a convolution integral. The contribution of the process noise  $w(t)$  over an interval of length  $t$  to the state  $a$  is:

$$\int_0^t g_a(\mu) w(t - \mu) d\mu, \quad (\text{A.14})$$

where  $g_a(\mu)$  is the weighting function (i.e., the inverse Laplace transform of the transfer function from  $w$  to  $a$ ). Table A.1 shows the three weighting functions that correspond to the transfer functions from  $w$  to each of the states in Figure A.2.



**Table A.1. Transfer and weighting functions.** The transfer functions from the input  $w(t)$  to each of the state variables. The inverse Laplace transform is referred to as the weighting function.

Transfer Functions	Weighting Functions
$G_{w \text{ to } a}(s) = \frac{\sqrt{2\sigma^2\beta}}{s + \beta}$	$g_a(t) = \sqrt{2\sigma^2\beta} e^{-\beta t}$
$G_{w \text{ to } v}(s) = \frac{\sqrt{2\sigma^2\beta}}{s(s + \beta)}$	$g_v(t) = \sqrt{2\sigma^2\beta} \frac{1}{\beta} (1 - e^{-\beta t})$
$G_{w \text{ to } x}(s) = \frac{\sqrt{2\sigma^2\beta}}{s^2(s + \beta)}$	$g_x(t) = \sqrt{2\sigma^2\beta} \frac{1}{\beta^2} (\beta t - 1 + e^{-\beta t})$

Using (A.14),  $E[a a]$  is computed as the mean-square value of the contribution of  $w(t)$ , over an interval of length  $\Delta t$ , to the state  $a$ .

$$E[a a] = E \left[ \int_0^{\Delta t} g_a(\mu) w(\Delta t - \mu) d\mu \int_0^{\Delta t} g_a(\nu) w(\Delta t - \nu) d\nu \right] \quad (\text{A.15a})$$

$$= \int_0^{\Delta t} \int_0^{\Delta t} g_a(\mu) g_a(\nu) E[w(\Delta t - \mu) w(\Delta t - \nu)] d\mu d\nu \quad (\text{A.15b})$$

Since  $w(t)$  is stationary, the expectation in (A.15b) equals  $E[w(\mu) w(\nu)]$ , which is the autocorrelation function  $R_w(\mu - \nu)$ . The autocorrelation function for white noise is the Dirac delta function  $\delta(\mu - \nu)$ .

$$E[a a] = \int_0^{\Delta t} \int_0^{\Delta t} g_a(\mu) g_a(\nu) E[w(\mu) w(\nu)] d\mu d\nu \quad (\text{A.16a})$$

$$= \int_0^{\Delta t} \int_0^{\Delta t} g_a(\mu) g_a(\nu) \delta(\mu - \nu) d\mu d\nu \quad (\text{A.16b})$$

If a function  $g(x)$  is continuous at  $x = \alpha$ , then over any interval that includes  $x = \alpha$ ,

$$\int g(x) \delta(x - \alpha) dx = g(\alpha), \quad (\text{A.17})$$

where  $\delta(x - \alpha)$  is the Dirac delta function at  $x = \alpha$  (Drake, 1967). Using (A.17), (A.16b) may be rewritten.

$$\text{E}[a a] = \int_0^{\Delta t} (g_a(\nu))^2 d\nu \quad (\text{A.18a})$$

$$= 2\sigma^2\beta \int_0^{\Delta t} e^{-2\beta\nu} d\nu \quad (\text{A.18b})$$

$$= \sigma^2 (1 - e^{-2\beta\Delta t}) \quad (\text{A.18c})$$

The other elements of  $Q$  are shown in Table A.2. Notice that  $Q$  depends on the time step,  $\Delta t$ , the mean-square value of the process noise,  $\sigma^2$ , and the time constant,  $\beta$ .

## A.2 Discrete-time Kalman Filter

Given the above model for the process by which  $\mathbf{x}$  evolves and some initial information about the state, a Kalman filter is used to generate models of the state at a particular time  $t$ ,  $f_{\mathbf{x}}(t)$ , incorporating new information when available. Table A.3 summarizes the Kalman filter algorithm and Table A.4 explains the terminology. The Kalman filter runs iteratively, first propagating the state estimate  $\hat{\mathbf{x}}_k$  and error covariance matrix  $P_k$  forward in time, producing an a priori estimate of the

**Table A.2. Elements of the Q matrix.**

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$$\begin{aligned} E[a a] &= \int_0^{\Delta t} \int_0^{\Delta t} 2\sigma^2 \beta e^{-\beta\mu} e^{-\beta\nu} \delta(\mu - \nu) d\mu d\nu \\ &= \sigma^2 (1 - e^{-2\beta\Delta t}) \end{aligned}$$

$$\begin{aligned} E[a v] &= \int_0^{\Delta t} \int_0^{\Delta t} 2\sigma^2 \beta e^{-\beta\mu} \frac{1}{\beta} (1 - e^{-\beta\nu}) \delta(\mu - \nu) d\mu d\nu \\ &= 2\sigma^2 \beta \left[ \frac{1}{2\beta^2} - \frac{1}{\beta^2} e^{-\beta\Delta t} + \frac{1}{2\beta^2} e^{-2\beta\Delta t} \right] \end{aligned}$$

$$\begin{aligned} E[a x] &= \int_0^{\Delta t} \int_0^{\Delta t} 2\sigma^2 \beta e^{-\beta\mu} \frac{1}{\beta^2} (\beta\nu - 1 + e^{-\beta\nu}) \delta(\mu - \nu) d\mu d\nu \\ &= 2\sigma^2 \beta \left[ \frac{1}{2\beta^3} - \frac{1}{\beta^2} \Delta t e^{-\beta\Delta t} - \frac{1}{2\beta^3} e^{-2\beta\Delta t} \right] \end{aligned}$$

$$\begin{aligned} E[v v] &= \int_0^{\Delta t} \int_0^{\Delta t} 2\sigma^2 \beta \frac{1}{\beta} (1 - e^{-\beta\mu}) \frac{1}{\beta} (1 - e^{-\beta\nu}) \delta(\mu - \nu) d\mu d\nu \\ &= 2\sigma^2 \beta \left[ \frac{1}{\beta^2} \Delta t - \frac{3}{2\beta^3} + \frac{2}{\beta^3} e^{-\beta\Delta t} - \frac{1}{2\beta^3} e^{-2\beta\Delta t} \right] \end{aligned}$$

$$\begin{aligned} E[v x] &= \int_0^{\Delta t} \int_0^{\Delta t} 2\sigma^2 \beta \frac{1}{\beta} (1 - e^{-\beta\mu}) \frac{1}{\beta^2} (\beta\nu - 1 + e^{-\beta\nu}) \delta(\mu - \nu) d\mu d\nu \\ &= 2\sigma^2 \beta \left[ \frac{1}{2\beta^4} + \frac{1}{2\beta^2} \Delta t^2 - \frac{1}{\beta^3} \Delta t - \frac{1}{\beta^4} e^{-\beta\Delta t} + \frac{1}{\beta^3} \Delta t e^{-\beta\Delta t} + \frac{1}{2\beta^4} e^{-2\beta\Delta t} \right] \end{aligned}$$

$$\begin{aligned} E[x x] &= \int_0^{\Delta t} \int_0^{\Delta t} 2\sigma^2 \beta \frac{1}{\beta^2} (\beta\mu - 1 + e^{-\beta\mu}) \frac{1}{\beta^2} (\beta\nu - 1 + e^{-\beta\nu}) \delta(\mu - \nu) d\mu d\nu \\ &= 2\sigma^2 \beta \left[ \frac{1}{3\beta^2} \Delta t^3 - \frac{1}{\beta^3} \Delta t^2 + \frac{1}{\beta^4} \Delta t - \frac{2}{\beta^4} \Delta t e^{-\beta\Delta t} + \frac{1}{2\beta^5} - \frac{1}{2\beta^5} e^{-2\beta\Delta t} \right] \end{aligned}$$


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state at time  $t_{k+1}$ ,  $\hat{\mathbf{x}}_{k+1}^-$ , and the corresponding error covariance,  $P_{k+1}^-$ . If available, new information is then incorporated, updating the estimate and error covariance at time  $t_{k+1}$  (Brown, 1983). The initial values of  $\hat{\mathbf{x}}$  and  $P$  which are required to start the cyclic algorithm are given by the problem statement.

At certain discrete times  $t_k$  (i.e., not necessarily for every integer value of  $k$ ), a measurement  $\mathbf{z}_k$  is taken.

$$\mathbf{z}_k = H \mathbf{x}_k + \mathbf{v}_k \quad (\text{A.19})$$

The observability matrix  $H$  defines which state variables are measured. Note that measurements are required to be linear combinations of the states.  $\mathbf{v}_k$  is the measurement noise.<sup>2</sup> The covariance of the measurement noise,  $R$ , is typically known from the problem statement (i.e., from the characteristics of the sensor or measurement process).

$$E[\mathbf{v}_k \mathbf{v}_i] = \begin{cases} R & \text{if } i = k \\ 0 & \text{otherwise} \end{cases} \quad (\text{A.20})$$

Using the twice-integrated, first-order Markov process from Section A.1.2 as an example, Table A.5 shows the observability matrix and measurement noise covariance matrix for two cases. In one case, only the output state  $x(t)$  is measured. In the second case, the states  $x(t)$  and  $v(t)$  are both measured.

If a measurement is absent at time  $t_k$ , the covariance of the measurement noise,  $R$ , is infinitely large and, therefore, the Kalman gain  $K$  will be zero. The expression for the state update reduces to  $\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^-$ , and the expression for the error covariance update reduces to  $P_k = P_k^-$ .

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<sup>2</sup> The measurement noise  $\mathbf{v}_k$  should not be confused with the velocity state  $v(t)$  in Figure A.2.

**Table A.3. Kalman filter algorithm (Gelb, 1974).**

I is the identity matrix.

Kalman Gain	$K = P_k^- H^T (H P_k^- H^T + R)^{-1}$
State Estimate Update	$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + K (\mathbf{z}_k - H \hat{\mathbf{x}}_k^-)$
Error Covariance Update	$P_k = (I - K H) P_k^-$
State Estimate Extrapolation	$\hat{\mathbf{x}}_{k+1}^- = \Phi \hat{\mathbf{x}}_k$
Error Covariance Propagation	$P_{k+1}^- = \Phi P_k \Phi^T + Q$

**Table A.4. Kalman filter terminology.**

Symbol	Definition
$\hat{\mathbf{x}}_k$	Kalman estimate for the state at time $t_k$ .
$\hat{\mathbf{x}}_k^-$	Propagation of the estimate prior to $\hat{\mathbf{x}}_{k-1}$ forward to time $t_k$ , incorporating new information.
$\mathbf{z}_k$	Measurement at time $t_k$ .
$K$	Kalman gain.
$P_k$	Error covariance matrix (i.e., uncertainty in the state estimate) at time $t_k$ .
$Q$	Covariance matrix for the process noise.
$R$	Covariance matrix for the sensor/measurment noise.
$\Phi$	State transition matrix.
$H$	Observability matrix.

**Table A.5. Measurement noise covariance matrix and observability matrix.**  $\sigma_{z_x}^2$  and  $\sigma_{z_v}^2$  are the expected errors in position and velocity measurements, respectively.

	$x(t)$ Measured	$x(t)$ and $v(t)$ Measured
$H$	$[1 \ 0 \ 0]$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
$R$	$\sigma_{z_x}^2$	$\begin{bmatrix} \sigma_{z_x}^2 & 0 \\ 0 & \sigma_{z_v}^2 \end{bmatrix}$

The model of the state, which is the purpose of employing the Kalman filter, is given by the multivariate normal probability density function, where the expected value of the PDF is given by the Kalman estimate  $\hat{\mathbf{x}}$  and the covariance matrix is given by the Kalman error covariance matrix  $P$ .  $S$  is the length of  $\mathbf{x}$  and  $|P|$  is the determinant of  $P$ .

$$f_{\mathbf{x}}(\mathbf{x}) = \frac{1}{(2\pi)^{\frac{S}{2}} |P|^{\frac{1}{2}}} e^{-\frac{1}{2}[(\mathbf{x}-\hat{\mathbf{x}})^T P^{-1}(\mathbf{x}-\hat{\mathbf{x}})]} \quad (\text{A.21})$$

### A.3 Stochastic Models for Aircraft Separation

Given a method for constructing a probabilistic model for the state of an aircraft at any time  $t$  (i.e, the Kalman filter and the model of the dynamics of the state), a stochastic model for the separation between two aircraft is sought. The

aircraft separation model will be used to calculate the probability that a conflict will occur between the two aircraft.

### A.3.1 Separation in a Single Dimension

Assume two aircraft, A and B, are flying along the same airway (i.e., the second aircraft is following the first aircraft), such that their positions may be described along a single dimension  $x$ . Let their positions at time  $t$  be independent, normally distributed random variables  $x_A$  and  $x_B$ , with expectations  $E[x_A]$  and  $E[x_B]$  and variances  $\sigma_{x_A}^2$  and  $\sigma_{x_B}^2$ . Figure A.3 illustrates this scenario. The distance between the two aircraft is a new random variable (RV)  $s$ , also normally distributed.

$$s = x_A - x_B \quad (\text{A.22})$$

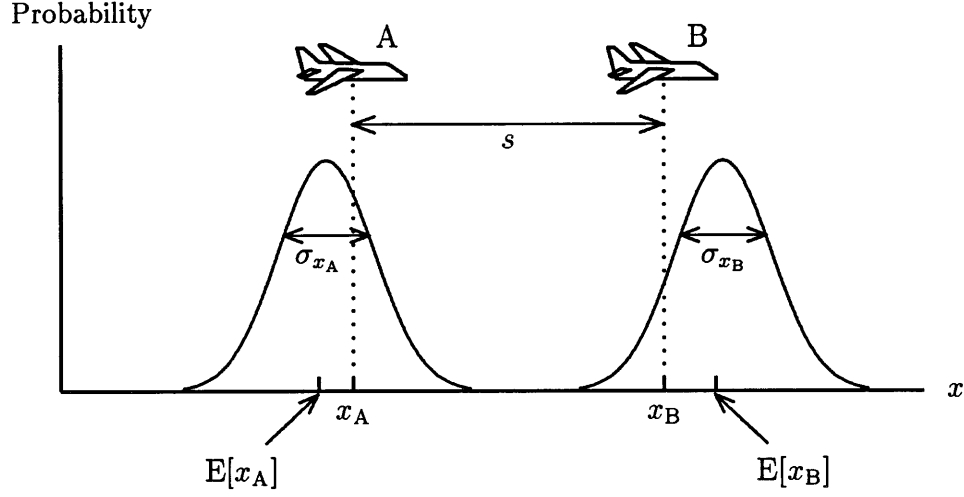
$$E[s] = E[x_A] - E[x_B]$$

$$\sigma_s^2 = \sigma_{x_A}^2 + \sigma_{x_B}^2$$

### A.3.2 Separation in Two Dimensions

If the two aircraft are separated in two dimensions (i.e., the aircraft are at the same altitude), their separation is more difficult to model, even under the assumption that the positions of each aircraft are normally distributed. Assume the aircrafts' positions are described by independent, bivariate normal random variables,  $\mathbf{x}_A = [x_A, y_A]^T$  and  $\mathbf{x}_B = [x_B, y_B]^T$ . Equation (A.23) defines the probability density for a bivariate normal random variable  $\mathbf{x}$ .

$$f_{\mathbf{x}}(\mathbf{x}) = \frac{1}{2\pi|P|^{\frac{1}{2}}} e^{-\frac{1}{2}[(\mathbf{x}-\hat{\mathbf{x}})^T P^{-1}(\mathbf{x}-\hat{\mathbf{x}})]} \quad (\text{A.23})$$



**Figure A.3. Aircraft separation in a single dimension.**  
The variances  $\sigma_{x_A}^2$  and  $\sigma_{x_B}^2$  may be unequal.

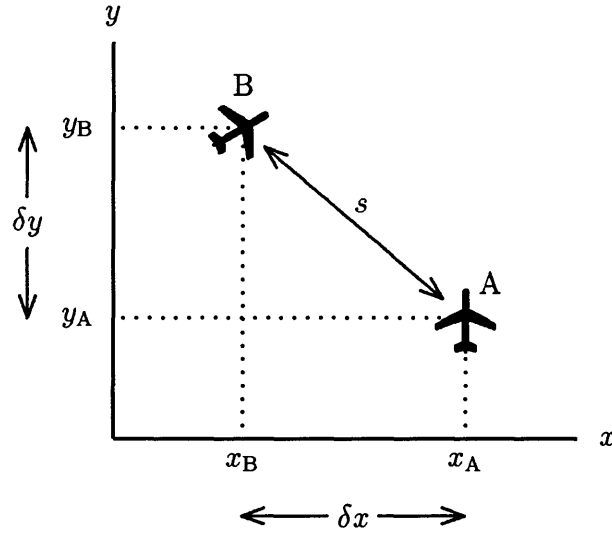
$\hat{\mathbf{x}}$  is the vector of expected values (i.e.,  $\hat{x} = E[x]$  and  $\hat{y} = E[y]$ ) and  $P$  is the covariance matrix.

$$\begin{aligned}
 P &= E[(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T] \\
 &= \begin{bmatrix} E[(x - \hat{x})^2] & E[(x - \hat{x})(y - \hat{y})] \\ E[(x - \hat{x})(y - \hat{y})] & E[(y - \hat{y})^2] \end{bmatrix} \\
 &= \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix}
 \end{aligned} \tag{A.24}$$

The variances  $\sigma_x^2$  and  $\sigma_y^2$  are the mean squared errors in the knowledge of  $x$  and  $y$ , respectively.  $\rho\sigma_x\sigma_y$  is the cross-correlation, with the correlation coefficient  $\rho$ .

Define two new random variables  $\delta x$  and  $\delta y$  which are the projections of the aircraft separation onto the  $x$  and  $y$  axes, respectively, as shown in Figure A.4.





**Figure A.4. Orthogonal components of aircraft separation.**

$\delta x$  and  $\delta y$  are also independent and normally distributed.

$$\delta x = x_A - x_B \quad (\text{A.25})$$

$$\delta y = y_A - y_B$$

The distance between the two aircraft is a new random variable  $s$ . The objective is to determine the probability density function (PDF) for  $s$ ,  $f_s$ , so that the probability of a conflict can be calculated. A conflict occurs when the separation  $s$  is less than the minimum separation criterion  $\theta$ . The probability of a conflict is computed by integrating the probability density function  $f_s$  over the range from  $s=0$  to  $s=\theta$ . Although  $\delta x$  and  $\delta y$  are normal random variables,  $s$  is not normally distributed because it is a non-linear function of  $\delta x$  and  $\delta y$ , as seen in Equation (A.26).

$$s = \sqrt{\delta x^2 + \delta y^2} \quad (\text{A.26})$$

$$= \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$

If  $\delta x$  and  $\delta y$  have zero means and equal variances, then  $s$  has a Rayleigh probability density function (PDF) (Papoulis, 1965). The Rayleigh PDF, defined in Equation (A.27), describes the likelihood of a circularly symmetric (i.e., equal variances), bivariate normal random variable taking on a value that is a distance  $s$  from the expectation. However,  $\delta x$  and  $\delta y$  cannot both have non-zero means, because they represent the separation between the two aircraft.

$$f_s(\acute{s}) = \frac{1}{\sigma^2} \acute{s} e^{-\frac{\acute{s}^2}{2\sigma^2}} \quad \acute{s} > 0 \quad (\text{A.27})$$

If  $\delta x$  and  $\delta y$  have equal variances and either  $\delta x$  or  $\delta y$  has a zero mean (and the other has a non-zero mean), then Papoulis (1965) offers a closed-form expression for the probability density function of  $s$ . Assume  $E[\delta y] = 0$  and  $E[\delta x] \neq 0$ .

$$f_s(\acute{s}) = \frac{\acute{s}}{\sigma^2} I_0\left(\frac{\acute{s} E[\delta x]}{\sigma^2}\right) e^{-\frac{\acute{s}^2 + E[\delta x]^2}{2\sigma^2}} \quad \acute{s} > 0 \quad (\text{A.28})$$

where the modified Bessel function  $I_0(\alpha)$  is defined as:

$$I_0(\alpha) = \frac{1}{2\pi} \int_0^{2\pi} e^{\alpha \cos \theta} d\theta \quad (\text{A.29})$$

Papoulis' approach can also be used to determine  $f_s$  when both  $\delta x$  and  $\delta y$  have non-zero expected values. Define a new coordinate system  $\{x', y'\}$ , translated and rotated with respect to the original system, such that the bivariate joint distribution describing the position of one aircraft is centered at the origin of the new system (i.e.,  $E[x'_A] = 0$  and  $E[y'_A] = 0$ ) and that for the other aircraft lies on the new  $x'$  axis (i.e.,  $E[x'_B] = [(E[x_A] - E[x_B])^2 + (E[y_A] - E[y_B])^2]^{\frac{1}{2}}$  and  $E[y'_B] = 0$ ). In the

new coordinate system,  $\delta x$  and  $\delta y$  remain normally distributed, and  $E[\delta y]=0$  and  $E[\delta x] \neq 0$ . Equation (A.28) may now be used as the probability density function for the distance  $s$  between the two aircraft. The probability of a conflict may be computed by integrating this distribution from  $s=0$  to  $s=\theta$ .

### A.3.3 A Numerical Method for Calculating the Probability of a Conflict

If the positions of the aircraft,  $\mathbf{x}_A$  and  $\mathbf{x}_B$ , are not normally distributed, or the variances are not equal, the probability of a conflict may be calculated using a numerical approach. The cumulative distribution function  $P_{s \leq}(s \leq \theta)$  for the probability that the separation  $s$  is less than a value  $\theta$  may be found by integrating the joint probability density function  $f_{\delta x \delta y}$  over the circular region of radius  $\theta$  centered around  $s=0$ .  $f_{\delta x \delta y} = f_{\delta x} f_{\delta y}$  because the random variables  $\delta x$  and  $\delta y$  are statistically independent.

$$P_{s \leq}(s \leq \theta) = \int_{-\theta}^{\theta} \int_{-\sqrt{\theta^2 - \delta \dot{x}^2}}^{\sqrt{\theta^2 - \delta \dot{x}^2}} f_{\delta x}(\delta \dot{x}) f_{\delta y}(\delta \dot{y}) d(\delta \dot{y}) d(\delta \dot{x}) \quad (\text{A.30})$$

where, for example,

$$f_{\delta x}(\delta \dot{x}) = \frac{1}{\sqrt{2\pi} \sigma_{\delta x}} e^{-\frac{(\delta \dot{x} - E[\delta x])^2}{2\sigma_{\delta x}^2}} \quad (\text{A.31})$$

If  $\theta$  is the minimum separation criterion, then  $P_{s \leq}(s \leq \theta)$  is the probability of a violation. Generalizing the approach to separation in three dimensions is straight forward, although vertical and horizontal aircraft separation requirements are typically treated separately.

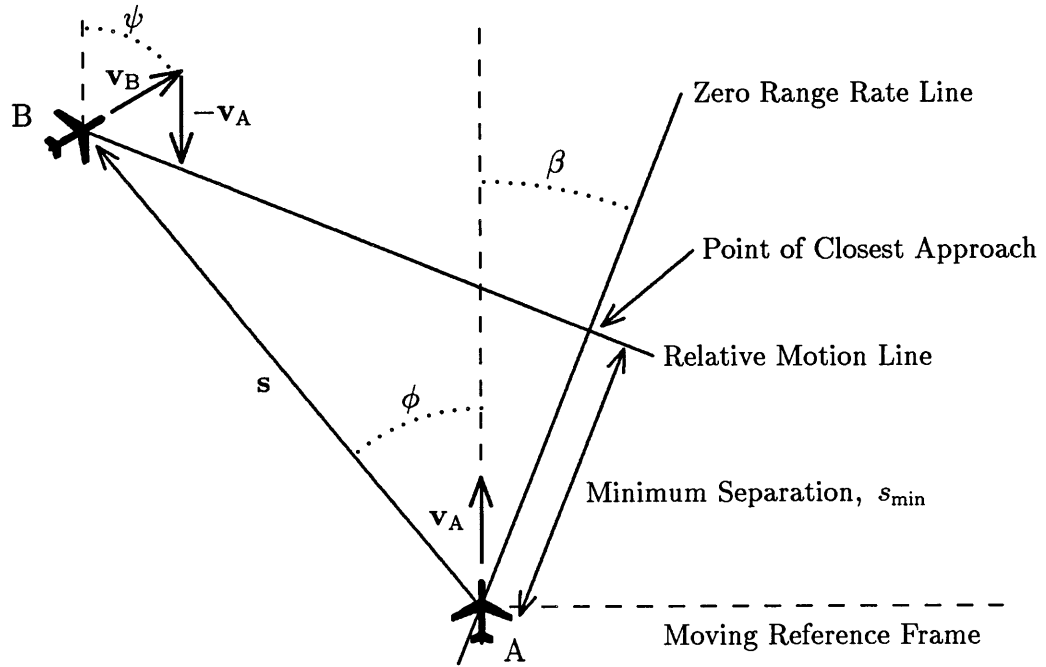


Figure A.5. Aircraft conflict geometry (Krozel, 1997).

#### A.3.4 Approximate Method for Calculating the Probability of a Conflict

The following method is borrowed from Krozel et al. (1997) and Krozel and Peters (1997). Consider two aircraft flying at constant and equal altitudes, each with no side-slip. Figure A.5 illustrates the encounter geometry.  $\phi$  is the instantaneous angle between aircraft A's velocity vector  $\mathbf{v}_A$  and the instantaneous position vector  $\mathbf{s}$ , locating aircraft B relative to aircraft A.  $s$  is the instantaneous distance between the aircraft (i.e., the magnitude of  $\mathbf{s}$ ).  $\psi$  is the relative heading of aircraft B, with respect to aircraft A.

The velocity of aircraft B relative to aircraft A,  $\mathbf{c} = \mathbf{v}_B - \mathbf{v}_A$ , may be decomposed into an instantaneous component along the relative position vector  $\mathbf{s}$ ,

which produces a time rate of change  $\frac{ds}{dt}$  in the aircraft separation and a tangential component. The *zero range rate line* is the set of points for which  $\frac{ds}{dt} = 0$ . Consider a moving reference frame attached to aircraft A. The relative motion of aircraft B is along a line perpendicular to the zero range rate line, and the *point of closest approach* is the intersection between this *relative motion line* and the zero range rate line. The minimum aircraft separation,  $s_{\min}$ , is called the *miss distance*. Depending on the geometry, the encounter will result in a conflict if  $|s_{\min}| \leq \theta$ , or no conflict otherwise.  $\theta$  is the required separation.

$\beta$  is the angle between aircraft A's velocity vector and the zero range rate line. From Krozel (1997):

$$\beta = \cos^{-1} \left[ \frac{v_B \sin \psi}{\sqrt{v_A^2 + v_B^2 - 2 v_A v_B \cos \psi}} \right] \quad (\text{A.32})$$

$\tau$  is the time to closest approach.

$$\tau = \frac{s \sin(\beta - \phi)}{\sqrt{v_A^2 + v_B^2 - 2 v_A v_B \cos \psi}} \quad (\text{A.33})$$

An expression for the miss distance  $s_{\min}$  may be found from analytic geometry, where  $s$  and  $\phi$  are measured at the same time.

$$s_{\min} = s \cos(\beta - \phi) \quad (\text{A.34})$$

Krozel's approach to predicting the probability that the current trajectory will lead to a conflict assumes that the positions of the aircraft are normally distributed

around measurements with known statistics, and the velocities are constants, normally distributed around measurements with known statistics. The uncertainty in the minimum separation results from the propagation of these uncertainties to the predicted time of closest approach. Given these models for the states of the aircraft at a point in time, Krozel derives the statistics of the miss distance. The expected value of the miss distance may be calculated using the expression:

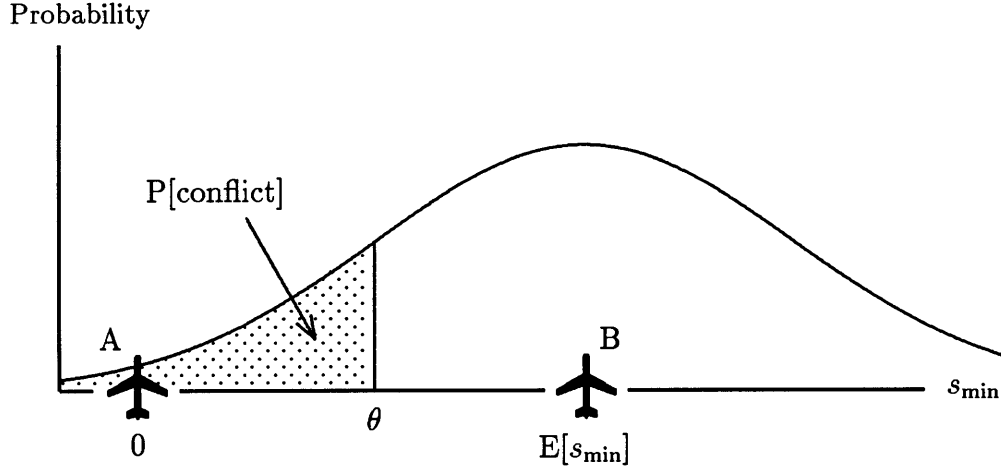
$$E[s_{\min}] = \frac{s_x c_y - s_y c_x}{\sqrt{c_x^2 + c_y^2}} \quad (\text{A.35})$$

where  $s_x$  and  $s_y$  are the components of the relative position vector  $\mathbf{s}$ .  $c_x$  and  $c_y$  are the components of the relative velocity vector  $\mathbf{c}$ . Assume that the expected errors in the measurements of each aircraft's position are  $\sigma_p^2$  along each axis, and the expected errors in the measurements of each aircraft's velocities are  $\sigma_v^2$  along each axis. Krozel shows the variance of the miss distance to be:

$$\sigma_{s_{\min}}^2 = 2\sigma_p^2 + 2\sigma_v^2\tau^2 \quad (\text{A.36})$$

Krozel also derives an alternate expression for the variance of the miss distance, if the aircraft velocities are measured in terms of speed and heading, rather than as two orthogonal speeds.

The minimum aircraft separation (i.e., miss distance),  $s_{\min}$ , is normally distributed with expectation  $E[s_{\min}]$  and variance  $\sigma_{s_{\min}}^2$ . Figure A.6 illustrates this probability density function. In two dimensions, a conflict occurs when  $|s_{\min}| \leq \theta$ . In three dimensions, the minimum separation being less than  $\theta$  may not constitute a conflict if the aircraft have sufficient vertical separation. Recall that the protected



**Figure A.6. Probability density function for minimum aircraft separation.** The probability of a conflict is calculated by integrating the probability density that aircraft B, at its point of closest approach (PCA), will be exactly a distance  $s_{\min}$  from aircraft A, over the region  $|s_{\min}| \leq \theta$ .

zone is not a sphere centered around the aircraft, but a hockey puck-shaped region, with the diameter being much larger than the thickness. The probability that a conflict will occur is calculated by integrating the PDF for the minimum aircraft separation,  $f_{s_{\min}}$ , over the range of separations which constitute a conflict.

$$P[\text{conflict}] = \frac{1}{\sqrt{2\pi} \sigma_{s_{\min}}} \int_{-\theta}^{\theta} e^{-\frac{(s-E[s_{\min}])^2}{2\sigma_{s_{\min}}^2}} ds \quad (\text{A.37})$$

If  $f_{s_{\min}}$  is scaled to the unit normal PDF, through a change of variable, then the probability of conflict may be evaluated using the standard error function, which is easier to implement and more computationally efficient than a numerical integration algorithm.

$$P[\text{conflict}] = \frac{1}{2} \operatorname{erf} \left( \frac{\theta + E[s_{\min}]}{\sqrt{2} \sigma_{s_{\min}}} \right) + \frac{1}{2} \operatorname{erf} \left( \frac{\theta - E[s_{\min}]}{\sqrt{2} \sigma_{s_{\min}}} \right) \quad (\text{A.38})$$

The standard error function:

$$\operatorname{erf}(\alpha) = \frac{2}{\sqrt{\pi}} \int_0^\alpha e^{-x^2} dx \quad (\text{A.39})$$

can be found by a lookup table or an efficient algorithm, which are readily available.



## Appendix B

### State Estimation with Imperfect Measurements

The derivation of the information value metric, in Sections 3.2 through 3.5, assumed that the information exactly revealed the condition of the state variable at the time of measurement. More generally, the measurement process may introduce a sensor error, or the state variable of interest may not be directly observable, requiring an estimate to be constructed from the observation of other states. These issues are considered presently.

#### B.1 Discrete State Variables and Measurements

The effect of sensor performance on the ability to estimate states is more clearly illustrated when the state is restricted to discrete values and transitions at discrete times. This discussion will be generalized to the continuous case in the following section. Assume that at a given discrete time  $t_k$  a relevant state variable  $x(t_k)$  may be characterized by one of  $N$  distinct values  $s_i$ .

$$x(t_k) \in \{s_1, s_2, \dots, s_N\} \tag{B.1}$$

Let the elements of the vector  $\pi(t_k)$  represent the probabilities that the state will take on each of the feasible values at the time  $t_k$ , based on the available information.  $\pi(t_k)$  replaces the continuous probability density function  $f_{x(t_k)}$  introduced earlier.

$$\pi_i(t_k) = P[x(t_k)=s_i] \quad (\text{B.2})$$

Assume that the state evolves as a Markov process, with transitions at the discrete times  $t_k$  governed by the time-invariant transition matrix  $\Phi$ .

$$\phi_{ij}(t_{k+1}, t_k) = P[x(t_{k+1})=s_i | x(t_k)=s_j] \quad (\text{B.3})$$

Without additional information, the state probability vector may be propagated according to (B.4), requiring initial knowledge of the state modeled by  $\pi(t_0)$ .

$$\pi(t_{k+1}) = \Phi(t_{k+1}, t_k) \pi(t_k) \quad (\text{B.4})$$

Estimation of the state is improved by incorporating measurements collected by a sensor at certain of the discrete times. Assume that measurements are restricted to be one of  $M$  discrete values  $y_m$ .

$$y(t_k) \in \{y_1, y_2, \dots, y_M\} \quad (\text{B.5})$$

The sensor performance is characterized by a conditional error probability.

$$p_{mi} = P[y(t_k)=y_m | x(t_k)=s_i] \quad (\text{B.6})$$

Assuming that a priori information yields an initial probability distribution  $\pi(t_0)$  describing knowledge of the state and that a sequence of observations  $Y_k$  is available, the state probability vector may be propagated and updated recursively.

$$Y_k = \{y(t_1), y(t_2), \dots, y(t_k)\} \quad (\text{B.7})$$

The conditional state probability vector at time  $t_k$ , prior to incorporating a measurement at  $t_k$ , is given by (B.8). The conditioning information is the a priori knowledge of the state and the sequence of observations omitting the observation at the current time.

$$\pi^-(t_k | \pi(t_0), Y_{k-1}) = \Phi(t_k, t_{k-1}) \pi(t_{k-1} | \pi(t_0), Y_{k-1}) \quad (\text{B.8a})$$

$$\pi_i^-(t_k | \pi(t_0), Y_{k-1}) = P[x(t_k) = s_i | \pi(t_0), Y_{k-1}] \quad (\text{B.8b})$$

New information is incorporated, according to Bayes' theorem, to update the conditional probabilities. The a posteriori conditional state probability vector will be:

$$\begin{aligned} \pi_i(t_k | \pi(t_0), Y_{k-1}, y(t_k) = y_m) &= P[x(t_k) = s_i | \pi(t_0), Y_{k-1}, y(t_k) = y_m] \\ &= \frac{\pi_i^-(t_k | \pi(t_0), Y_{k-1}) p_{mi}}{\sum_{j=1}^N \pi_j^-(t_k | \pi(t_0), Y_{k-1}) p_{mj}} \end{aligned} \quad (\text{B.9})$$

If a measurement is not received at  $t_k$ , the state probability vector is updated by:

$$\pi(t_k | \pi(t_0), Y_k) = \pi^-(t_k | \pi(t_0), Y_{k-1}) \quad (\text{B.10})$$

Assume that at time  $t_k$  a decision maker must select an action  $a(t_k)$  from a set of  $A$  available actions.

$$a(t_k) \in \{a_1, a_2, \dots, a_A\} \quad (\text{B.11})$$

Decision problems offer great variety in the timing of the decision. In this discussion, the decision is assumed to be made at a specific time  $t_k$ . Other problems might allow the decision to be made at any time prior to a critical decision time, or require the decision to be made continuously, for example in monitoring tasks.

Assume that a known reward structure assigns an outcome  $r_{ij}$  when  $x(t_k) = s_i$  and  $a(t_k) = a_j$ . In general, the reward structure may be time varying; in this discussion it is assumed to be constant. Further assume that the decision maker's objective is to maximize the conditional expected payoff which will result from the chosen action. The conditioning is based on the knowledge of the state provided by the initial knowledge  $\pi(t_0)$  and the observations  $Y_k$ . This assumption implies that the decision maker applies the measurements in an optimal manner (i.e., according to Bayes' rule), knows the correct model for the sensor's conditional error probabilities, and knows the correct state transition matrix.

A cost  $C$  is assessed for each observation. Therefore, the decision maker must first decide whether or not to acquire new information. This decision must be made before the content of the information is known to the decision maker. Therefore, the information seeking decision must consider all possible observation results as well as all possible conditions of the state.

With no additional information at time  $t_k$ , the expected outcome  $R$  if  $a(t_k) =$

$a_j$  is:

$$\mathbb{E}[R | \pi(t_0), Y_{k-1}, a(t_k) = a_j] = \sum_{i=1}^N \pi_i^-(t_k | \pi(t_0), Y_{k-1}) r_{ij} \quad (\text{B.12})$$

The assumed decision policy specifies the  $a_j$  which maximizes the expected outcome.

$$\mathbb{E}[R | \pi(t_0), Y_{k-1}] = \max_j \sum_{i=1}^N \pi_i^-(t_k | \pi(t_0), Y_{k-1}) r_{ij} \quad (\text{B.13})$$

$Y_{k-1}$  represents the set of measurements that is available prior to time  $t_k$ .

To evaluate the impact of a measurement at time  $t_k$ , prior to knowing the content of the measurement, the expected outcome must be expressed as an expectation over both the possible states and possible measurements.

$$\mathbb{E}[R | \pi(t_0), Y_{k-1}, y(t_k)] = \sum_{m=1}^M \mathbb{P}[y(t_k) = y_m] \left\{ \max_j \sum_{i=1}^N \pi_i(t_k | \pi(t_0), Y_{k-1}, y(t_k) = y_m) r_{ij} \right\} \quad (\text{B.14})$$

The probability that the measurement will take on each feasible value may be calculated from:

$$\begin{aligned} \mathbb{P}[y(t_k) = y_m] &= \sum_{i=1}^N \mathbb{P}[y(t_k) = y_m | x(t_k) = s_i] \mathbb{P}[x(t_k) = s_i] \\ &= \sum_{i=1}^N p_{mi} \pi_i^-(t_k | \pi(t_0), Y_{k-1}) \end{aligned} \quad (\text{B.15})$$

Applying (B.9) and (B.15) to (B.14), the expected outcome may be rewritten.

$$\begin{aligned} \mathbb{E}[R | \pi(t_0), Y_{k-1}, y(t_k)] &= \sum_{m=1}^M \left[ \sum_{l=1}^N p_{ml} \pi_l^-(t_k | \pi(t_0), Y_{k-1}) \right] \times \\ &\quad \max_j \left[ \sum_{i=1}^N \frac{\pi_i^-(t_k | \pi(t_0), Y_{k-1}) p_{mi}}{\sum_{l=1}^N \pi_l^-(t_k | \pi(t_0), Y_{k-1}) p_{ml}} r_{ij} \right] \end{aligned} \quad (\text{B.16})$$

Although the measurement is not known to the decision maker,  $y(t_k)$  is shown as conditioning information on the left of (B.16) as a reminder that the expectation is being taken over the measurement as well as the state. The decision maker's information seeking rule will be to purchase the measurement  $y(t_k)$  when  $E[R | \pi(t_0), Y_{k-1}, y(t_k)] - C > E[R | \pi(t_0), Y_{k-1}]$ . The expected value of the information is  $E[R | \pi(t_0), Y_{k-1}, y(t_k)] - E[R | \pi(t_0), Y_{k-1}]$ .

## B.2 Continuous State Variables and Measurements

The approach to optimally (minimum variance Bayesian estimate) incorporating information into the estimate of a state variable  $x$  when the state and measurements are continuous numbers is parallel in form to the previous case, in which the state and measurements were restricted to finite sets of discrete feasible values. The principle relationships are presented without repeating the tutorial, to illustrate their commonality with the discrete case.

Assume a model of the random process  $x$  is known, replacing the state transition matrix from the discrete-state case. The observed state trajectory  $x(t)$  is one sample realization from the ensemble that defines  $x$ . Assume the state at a time  $t_0$  is known deterministically:  $x(t_0) = x_0$ .

Knowledge of the random process and  $x(t_0)$  allows an a priori model of the state at a later time  $t_1$  to be determined:  $f_{x(t_1)|x(t_0)}$ .  $x(t_1)$  is a continuous random variable. This PDF replaces the vector of probabilities  $\pi(t_1)$  in Equation (B.2). Knowledge of the sensor performance is modeled by a conditional error probability

density function:  $f_{z(t_1)|x(t_1)}$ . Assuming  $x(t_1) = x_1$ , the measurement  $z_1$  that will be realized is distributed according to  $f_{z(t_1)|x(t_1)}(z_1 | x_1)$ . This PDF replaces the probability mass function  $p_{mi}$ .

The model, based on the initial information, for the measurement that will be received is:

$$f_{z(t_1)|x(t_0)}(z_1 | x_0) = \int_{\hat{x}} f_{z(t_1)|x(t_1)}(z_1 | \hat{x}) f_{x(t_1)|x(t_0)}(\hat{x} | x_0) d\hat{x} \quad (\text{B.17})$$

Note that since  $z(t_1)$  and  $x(t_0)$  are statistically independent,  $f_{z(t_1)|x(t_1)}$ , the probability density function describing sensor performance, is not conditioned on the initial state. Also note the similarity with (B.15).

Given that the measurement  $z(t_1)$  is  $z_1$ , an a posteriori model of the state at  $t_1$  may be determined by Bayes' rule.

$$f_{x(t_1)|z(t_1), x(t_0)}(x_1 | z_1, x_0) = \frac{f_{z(t_1)|x(t_1)}(z_1 | x_1) f_{x(t_1)|x(t_0)}(x_1 | x_0)}{f_{z(t_1)|x(t_0)}(z_1 | x_0)} \quad (\text{B.18})$$

The notation in (B.18),  $f_{x(t_1)|z(t_1), x(t_0)}(x_1 | z_1, x_0)$ , implies that the measurement at  $t_1$  is known to be  $z_1$ . Therefore, to compare the a priori and a posteriori models of  $x(t_1)$ , prior to knowing  $z(t_1)$ , requires taking the expectation over the feasible measurements, as was done before.

“Have you ever thought, not only about the airplane but about whatever man builds, that all of man’s industrial efforts, all his computations and calculations, all the nights spent working over droughts and blueprints, invariably culminate in the production of a thing whose sole and guiding principle is the ultimate principle of simplicity?

...In anything at all, perfection is finally attained not when there is no longer anything to add, but when there is no longer anything to take away, when a body has been stripped down to its nakedness.”

Yvon Chouinard – climber, environmentalist.



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## Colophon

This thesis was typeset in the *Computer Modern* family of fonts using the T<sub>E</sub>X document preparation environment. Graphics were drawn using P<sub>I</sub>CT<sub>E</sub>X, and plots were created using *Matlab* (The MathWorks, Natick, MA).

“Well, we knocked the bastard off!”

Sir Edmund Hillary – to George Lowe upon returning to high camp after first climbing Sagar-matha (Mount Everest) with Tenzing Norgay Sherpa, 1953.